

MINIMUM-ENERGY CONSUMPTION OF AN INDUCTION MOTOR OPERATING IN DYNAMIC REGIME

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Abstract: *A new control strategy for Induction Motor (IM) operating at variable speed and torque is proposed in this paper. The entire purpose is to introduce a new energy optimization approach using dynamic constraint of rotor variables under Rotor Field Oriented Control (RFOC). Such control strategy is based on a designed cost function given as a weighting sum of stored energy, coil losses, and mechanic power. The proposed algorithm is devoted to establish the optimal dynamic of rotor flux versus the motor speed by taking into account two constraints linked to transient regimes of rotor flux and speed. From such algorithm, a time-varying rotor flux values can be determined which optimizing IM energy. Aiming to check its validity, the proposal was implemented in a RFOC of a 1.5 kW laboratory IM. Regarding to recent works and to the presented algorithm results, better performances can be obtained since lowest energy consumption of the IM is reached in dynamic regime.*

Keywords: Induction motor, optimal control, energy minimization, field oriented control, dynamic regime.

1. Introduction

Some of the largest opportunities to save energy and reduce operating costs in buildings and industrial facilities come from optimizing electric motor systems. In general, the most part of electricity assumed flows through motors mainly induction motors. The IM is the dominant technology used today due to its high performance, its high reliability, and its speed and torque capabilities. Many control strategies of minimizing energy consumption of IM were proposed in the literature; most of them lead to increase the IM efficiency [11] and [12].

The conventional RFOC method operating at constant rotor flux norm fixed at its standard level provides maximal efficiency when the system operates at its standard operating point. Far from this, the machine's efficiency decreases; it can result from a torque magnitude change. Thus other modes of flux operation are required in order to reach system with optimal performances.

Certain works emphasised a developing and implementing of the RFOC and minimum-energy

approach simultaneously [7], [10], [14], and [15]. Tr. Munteanu et al. [8] used the matrix Riccati differential equation to solve a quadratic performance criterion that involves energy in both inertia and motor windings. They gave an online numerical solution for the IM drive. Aiyuan Wang et al. [9] proposed a performance criteria index containing copper and iron losses. The IM model took into account the iron loss resistance which was introduced in the expression of the different currents. They proposed an online numeric solution of the optimal rotor flux. C. Canudas de Wit et al. [6] considered a convex energy cost function including the stored magnetic energy and the coil losses. They developed a nonlinear Euler-Lagrange equation, from which an optimal flux norm trajectory can be derived. The obtained equation was unsolvable for an arbitrary torque. They proposed a suboptimal analytical solution aligned with a constant torque operation.

Keeping on this framework, an original technique leading to determine the optimal rotor flux in dynamic regime is proposed. The method use a reduced model of the IM in a turning (d,q) reference frame [6]. The proposed optimum-energy method is based on the optimal control theory focus on minimizing a cost function given as an integral of a weighted sum of energy and power of the IM. The reduced model is composed by the dynamics of motor speed and rotor flux and it is introduced as dynamic constraints in the Optimal Control Problem (OCP). The minimum of the cost function is subjected to those two dynamics constraint and This OCP defined as a minimum-energy approach is depending on two state variables: rotor flux and motor speed and two control variables: the rotor flux and the torque currents. The task in this OCP can be simplified to find the optimal rotor flux that provides the lowest IM's energy consumption along the given motor speed range.

To solve this problem, an important mathematical background is needed and dealing with a high complexity level of calculus. Some of the useful tools are the Euler-Lagrange equations [2]

By applying the Euler-Lagrange equation to the proposed OCP a system of nonlinear differential equations is established and by means of analytical resolution in the case of the steady state operation and a time-varying solution is given. But in the case of the transient regime, a numerical resolution is imposed and the solution is presented by a recursive formula. Both of the operation modes, the solutions provide a minimum-energy rotor flux trajectories. By implementing this solution as a reference to the rotor flux closed-loop of the RFOC, a minimum-energy consumption of the IM is registered.

In this paper, the second section is devoted to describe the dynamic model of the IM from which the full-order model then the reduced one and the energy model are developed. The third section is dedicated to present the OCP in the case of a minimum-energy approach. In the same section, the resolution of the OCP is performed by the help of the Euler-Lagrange equation. By choosing the mechanical operation modes, minimum-energy rotor flux trajectories are achieved. In section four, a deadbeat rotor flux controller is presented. In the fifth section, the Optimal Rotor Flux Oriented Control (ORFOC) is explained. The sixth section includes simulation results of the ORFOC compared to those obtained in the former studies which are experimentally proved.

2. Model of IM

2.1 Full-order dynamic model of IM

The full-order dynamic model of an IM viewed from the synchronous rotating reference frame is given by (1) to (3) [1, page 501]:

$$\begin{cases} \dot{I}_s^{(d,q)} = -\left(\gamma I + \left(\dot{\rho} + p\Omega\right)J\right)I_s^{(d,q)} + \gamma(aI - p\Omega)\Phi_r^{(d,q)} + \frac{V_s^{(d,q)}}{\sigma L_s} & (1) \\ \dot{\Phi}_r^{(d,q)} = -\left(aI + \rho J\right)\Phi_r^{(d,q)} + bI_s^{(d,q)} & (2) \\ \dot{\Omega} = -\frac{\tau_l}{J_m} + \frac{y}{J_m} & (3) \end{cases}$$

$$\text{where } I_s^{(d,q)} = \begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix}$$

$$, \Phi_r^{(d,q)} = \begin{bmatrix} \Phi_{rd} \\ \Phi_{rq} \end{bmatrix}, V_s^{(d,q)} = \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and}$$

$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, when I : is the 2x2 unit matrix and J : is the 2X2 skew symmetric matrix.

with $\omega_s = \dot{\rho}$: the sleep frequency, $\omega_r = p\Omega$: the electrical rotor speed; $\dot{I}_{sd} = \frac{d}{dt}I_{sd}$, $\dot{I}_{sq} = \frac{d}{dt}I_{sq}$: are the differential of magnetizing and torque currents, respectively; V_{sd} , V_{sq} : the stator voltage in d-q axis; Φ_{rd} , Φ_{rq} : rotor fluxes in d-q axis, p : pairs poles number, y : is the electromagnetic torque $y = c\left(I_s^{(d,q)}\right)^T J\Phi_r^{(d,q)}$;

and

$$\gamma = \frac{1}{\sigma L_s} \left(R_s + \frac{M^2}{L_r^2} R_r \right), \quad \eta = \frac{M}{\sigma L_s L_r}, \quad a = \frac{R_r}{L_r}, \quad b = aM,$$

$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

,when R_r and R_s are the rotor and stator resistances;

L_r and L_s : The rotor and the stator inductance,

respectively; M is the magnetizing inductance. J_m : is the total moment of inertia of the rotor and fly-wheel masses reduced to the motor shaft; τ_l : is the load torque.

τ_l is modelled as a function of rotor speed, as found in many industrial applications. For many mechanical systems, τ_l can be modelled proportional to the motor speed, as follows:

$$\tau_l = K_l \Omega \quad (4)$$

with K_l : is the load torque constant.

2.2. Reduced Model

The stator current is taking as an input control of the system. A high gain control current loop is chosen in order to simplify the optimization algorithm efficiency [1, page 494]. Such choice permits to use a reduced order current fed IM model; the current loop is given as [6]:

$$V_s^{(d,q)} = \frac{\sigma L_s}{\varepsilon} \left(U - I_s^{(d,q)} \right) \quad (5)$$

where $0 < \varepsilon \ll 1$ and U is the new machine control variable.

By considering this control, the reduced model can be obtained using the ‘‘singular perturbation’’ theorem [15]. This implies from (5), $I_s^{(d,q)} = U$. And from (1), (2) and (3), a reduced model of the IM is built as follows:

$$\begin{cases} \dot{\Phi}_r^{(d,q)} = -\left(aI + \dot{\rho}J\right)\Phi_r^{(d,q)} + bU \\ \dot{\Omega} = -\frac{K_l}{J_m}\Omega + \frac{cU^T J\Phi_r^{(d,q)}}{J_m} \end{cases} \quad (6)$$

$$\text{with } U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

2.3. Energy model of the IM

The instantaneous active power of an IM in rotating dq -frames is given by:

$$P_a = \frac{3}{2} \left(I_s^{(d,q)} \right)^T I_s^{(d,q)} \quad (7)$$

From equation (1), the input power is given by:

$$P_a = \frac{3}{2} \sigma L_s \left(\dot{I}_s^{(d,q)} \right)^T \left(I_s^{(d,q)} \right) + \gamma \left(I_s^{(d,q)} \right)^T \left(I_s^{(d,q)} \right) - \eta \left(\Phi_r^{(d,q)} \right)^T I_s^{(d,q)} + \eta P \Omega \left(\Phi_r^{(d,q)} \right)^T J I_s^{(d,q)} \quad (8)$$

The relation between the stator and rotor currents can be given as follows:

$$I_r^{(d,q)} = \frac{1}{L_r} \left(\Phi_r^{(d,q)} + M I_s^{(d,q)} \right) \quad (9)$$

The instantaneous active power is then given by:

$$P_a = \frac{3}{2} \left(\sigma L_s \left(\dot{I}_s^{(d,q)} \right)^T I_s^{(d,q)} + \frac{1}{L_r} \left(\dot{\Phi}_r^{(d,q)} \right)^T \Phi_r^{(d,q)} \right) + \frac{3}{2} \left(R_s \left(I_s^{(d,q)} \right)^T I_s^{(d,q)} + R_r \left(I_r^{(d,q)} \right)^T I_r^{(d,q)} \right) + \Omega y \quad (10)$$

Finally the active power expression is given as:

$$P_a = \frac{\partial}{\partial t} W_L + P_J + P_m \quad (11)$$

By means of a Field-Oriented Control drive:

$$\Phi_r^{(d,q)} = \begin{bmatrix} \Phi_r \\ 0 \end{bmatrix}, \text{ the derivate of the stored magnetic given}$$

in (11) can be expressed from (10) as follows:

$$\frac{\partial W_L}{\partial t} = \frac{3}{2} \left(\sigma L_s \left(\dot{I}_s^{(d,q)} \right)^T I_s^{(d,q)} + \frac{1}{L_r} \dot{\Phi}_r \Phi_r \right) \quad (12)$$

This yield the stored magnetic energy of the induction machine, given as follows:

$$W_L = \frac{3}{2} \left(\frac{\sigma L_s}{2} (u_1^2 + u_2^2) + \frac{1}{2L_r} \Phi_r^2 \right) \quad (13)$$

is the stored magnetic energy of the induction machine, and the total copper losses:

From equation (10), we can also assign:

$$P_J = \frac{3}{2} \left(R_s \left(I_s^{(d,q)} \right)^T I_s^{(d,q)} + R_r \left(I_r^{(d,q)} \right)^T I_r^{(d,q)} \right) \quad (14)$$

as the total copper losses.

By using equations (2) and (9), those losses can be expressed with respect to U and Φ_r as follows:

$$P_J = \frac{3}{2} \left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) (u_1^2 + u_2^2) - \frac{3}{2} \frac{R_r}{L_r^2} \Phi_r^2 - \frac{3}{2} \frac{1}{L_r} \frac{d}{dt} \Phi_r^2 \quad (15)$$

$$\text{Since the IM drive is under RFOC, } \Phi_r^{(d,q)} = \begin{bmatrix} \Phi_r \\ 0 \end{bmatrix},$$

and the mechanical power of the IM rotor can be expressed as:

$$P_m = \Omega y \quad (16)$$

In terms of rotor variables and torque current I_{sq} we obtain:

$$P_m = \frac{3}{2} p \left(\frac{M}{L_r} \right) \Phi_r u_2 \Omega \quad (17)$$

3. The optimal control problem

An optimal control problem is based on minimizing a cost function. In this case the cost function can be given as the integral of an index $f(I_{sd}, I_{sq}, \Phi_r, \Omega)$, given as follows:

$$J_r = \int_0^T f(I_{sd}, I_{sq}, \Phi_r, \Omega) dt \quad (18)$$

The index corresponds to the weighted sum:

$$f(I_{sd}, I_{sq}, \Phi_r, \Omega) = \alpha_1 W_L + \alpha_2 P_J + \alpha_3 P_m \quad (19)$$

The weighting factors $\alpha_{i=1,2,3}$ are used to scale power-energy combined convex criteria terms defined above. Minimizing the cost function consists on minimizing the magnetic energy W_L which corresponds, close to the rated operating point, to maximize the power factor. And it consists, also, on minimizing losses that increase the machine efficiency. By using (10), (12), (18) and (20), the cost function is given as follows:

$$J_r = \frac{3}{2} \int_0^T \left[\alpha_1 \left(\frac{\sigma_s}{2} (u_1^2 + u_2^2) + \frac{1}{2L_r} \Phi_r^2 \right) + \alpha_2 \left(\left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) (u_1^2 + u_2^2) - \frac{R_r}{L_r^2} \Phi_r^2 \right) - \frac{\alpha_2}{L_r} \frac{d}{dt} (\Phi_r^2) + \alpha_3 \left(p \frac{M}{L_r} \right) \Phi_r u_2 \Omega \right] dt \quad (20)$$

The integral $\int_0^T \left(-\frac{1}{L_r} \frac{d}{dt} (\Phi_r^2) \right) dt = \frac{1}{L_r} (\Phi_r^2(0) - \Phi_r^2(T))$

has no effect on the optimizing problem and can be omitted it from the integral.

And the system described in (6) can be rewritten with respect to the Field orientation and to the new vector

control variable $U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ as follows:

$$\begin{cases} \dot{\Phi}_r = -a \Phi_r + b u_1 \\ \dot{\Omega} = -\frac{K_l}{J_m} \Omega + \frac{c u_2 \Phi_r}{J_m} \end{cases} \quad (21)$$

3.1 Minimum-Energy approach

By using the reduced model given by the system (21) and the cost function given by the equation (20), an optimal control problem can be situated and presented as follows:

Find the optimal control variables u_1^* and u_2^* that minimize the following cost function:

$$J_r = \int_0^T (r_1 u_1^2 + r_2 u_2^2 + q_1 \Phi_r^2 + q_2 \Phi_r u_2 \Omega) dt \quad (22)$$

Subjected to the dynamic constraints given in the system (21).

where the weighting factors r_1 , r_2 , q_1 and q_2 must be positives.

$$r_1 = \frac{3}{4} \sigma_s \alpha_1 + \frac{3}{2} \left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) \alpha_2, \quad r_2 = r_1,$$

$$q_1 = \frac{3}{4} \frac{\alpha_1}{L_r} - \frac{3}{2} \frac{R_r}{L_r^2} \alpha_2, \quad q_2 = \frac{3}{2} p \frac{M}{L_r} \alpha_3$$

$\alpha_{i=1,2,3}$ are chosen so that q_1 is positive.

Otherwise, the task is to find an admissible control trajectory $U^* = \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix}$ generating the corresponding state

trajectory Φ_r^* defined as the optimal rotor flux to provide minimum of the cost function presented in equation (22).

The optimal control problem as being presented has the object of finding the optimized values of u_1^* and u_2^* which are defined as follows:

$$\begin{cases} u_1^* = \frac{1}{b} \left(\dot{\Phi}_r^* + a \Phi_r^* \right) \\ u_2^* = \frac{J_m}{c \Phi_r^*} \left(\dot{\Omega}^* + \frac{K_l}{J_m} \Omega^* \right) \end{cases} \quad (23)$$

By replacing the expression of u_1 and u_2 from the system (21) in the cost function in (22), one obtains:

$$J_r = \int_0^T \left(r_1 \left(\frac{1}{b} \left(\dot{\Phi}_r + a \Phi_r \right) \right)^2 + r_2 \left(\frac{J_m}{c \Phi_r} \left(\dot{\Omega} + \frac{K_l}{J_m} \Omega \right) \right)^2 + q_1 \Phi_r^2 + q_2 \Phi_r \left(\frac{J_m}{c \Phi_r} \left(\dot{\Omega} + \frac{K_l}{J_m} \Omega \right) \right) \Omega \right) dt \quad (24)$$

This yields:

$$J_r = \int_0^T \left(\lambda_1 \dot{\Phi}_r^2 + \lambda_2 \frac{\dot{\Omega}^2}{\Phi_r^2} + \lambda_3 \Phi_r \dot{\Phi}_r + \left(\frac{\lambda_4}{\Phi_r^2} + \lambda_5 \right) \Omega \dot{\Omega} + \lambda_6 \Phi_r^2 + \left(\frac{\lambda_7}{\Phi_r^2} + \lambda_8 \right) \Omega^2 \right) dt \quad (25)$$

with,

$$\lambda_1 = \frac{r_1}{a^2 M^2}, \quad \lambda_2 = \frac{r_2 J_m^2}{c^2}, \quad \lambda_3 = \frac{2r_1}{a M^2}, \quad \lambda_4 = \frac{2r_2 J_m K_l}{c^2},$$

$$\lambda_5 = \frac{q_2 J_m}{c}, \quad \lambda_6 = \frac{r_1}{M^2} + q_1, \quad \lambda_7 = \frac{r_2 K_l^2}{c^2}, \quad \lambda_8 = \frac{q_2 K_l}{c}.$$

$\lambda_{i=1 \dots 8}$ must be positive constants defined accordingly and consequently to the condition on the constants r_1 , r_2 , q_1 and q_2 .

3.1.1 Euler-Lagrange equation

The integral given in (22) can be expressed as follows:

$$J_r = \int_0^T L(\dot{\Phi}_r, \Phi_r, \dot{\Omega}, \Omega) dt \quad (26)$$

and can be solved with *Euler-Lagrange equation* [3], [4],[5] with respect to the following condition: This integral has an absolute minimum Ω^* and Φ_r^* , if their trajectory satisfies the following conditions:

$$\frac{\partial}{\partial \Phi_r} \left(L(\dot{\Phi}_r, \Phi_r, \dot{\Omega}, \Omega) \right) - \frac{\partial}{\partial t} \left(\frac{\partial}{\partial \dot{\Phi}_r} \left(L(\dot{\Phi}_r, \Phi_r, \dot{\Omega}, \Omega) \right) \right) = 0 \quad (27)$$

and

$$\frac{\partial}{\partial \Omega} \left(L(\dot{\Phi}_r, \Phi_r, \dot{\Omega}, \Omega) \right) - \frac{\partial}{\partial t} \left(\frac{\partial}{\partial \dot{\Omega}} \left(L(\dot{\Phi}_r, \Phi_r, \dot{\Omega}, \Omega) \right) \right) = 0 \quad (28)$$

3.2. Development of the strategy

3.2.1. Development of the Euler-Lagrange equation

Aiming to solve the equations (27) and (28) and by using the expression of the cost function in (25) The condition (27) becomes:

$$-2\lambda_1 \ddot{\Phi}_r - 2\lambda_2 \frac{\dot{\Omega}^2}{\Phi_r^3} - \frac{2\lambda_4}{\Phi_r^3} \Omega \dot{\Omega} + 2\lambda_6 \dot{\Phi}_r - \frac{2\lambda_7}{\Phi_r^3} \Omega^2 = 0 \quad (29)$$

Using (28) and regarding to (25), yields:

$$2 \left(\lambda_7 \frac{1}{\Phi_r^2} + \lambda_8 \right) \Omega + 4 \frac{\lambda_2}{\Phi_r^3} \dot{\Phi}_r \dot{\Omega} + 2 \frac{\lambda_4}{\Phi_r^3} \dot{\Phi}_r \Omega - 2 \frac{\lambda_2}{\Phi_r^2} \ddot{\Omega} = 0 \quad (30)$$

From (29) and (30), the energy optimization problem is described by a second order differential equation:

$$\begin{cases} \ddot{\Phi}_r = -a_0 \frac{\dot{\Omega}^2}{\Phi_r^3} - a_1 \frac{\Omega}{\Phi_r^3} \dot{\Omega} + a_2 \dot{\Phi}_r - a_3 \frac{\Omega^2}{\Phi_r^3} \\ \ddot{\Omega} = \frac{2}{\Phi_r} \dot{\Phi}_r \dot{\Omega} + a_4 \frac{\Omega}{\Phi_r} \dot{\Phi}_r + a_5 \Omega + a_6 \Phi_r^2 \Omega \end{cases} \quad (31)$$

where,

$$a_0 = \frac{\lambda_2}{\lambda_1}, a_1 = \frac{\lambda_4}{\lambda_1}, a_2 = \frac{\lambda_6}{\lambda_1}, a_3 = \frac{\lambda_7}{\lambda_1}, a_4 = \frac{\lambda_4}{\lambda_2}, a_5 = \frac{\lambda_7}{\lambda_2}, a_6 = \frac{\lambda_8}{\lambda_2}$$

3.2.2. Linear Time-varying motor speed

In order to obtain an accelerate mode (transient regime), the motor speed can be chosen as follows:

$$\Omega^* = x_2 = c_0 t + c_1 \quad (32)$$

with $c_0 > 0$.

As a consequence the second equation in the system (32) has no physical significance and can be skipped. Taking into account the equation (32), the first equation in system (31) becomes:

$$\ddot{\Phi}_r \Phi_r^3 - a_2 \Phi_r^4 = -(\gamma_1 t^2 + \gamma_2 t + \gamma_3) \quad (33)$$

with $\gamma_1 = a_3 c_0^2$, $\gamma_2 = a_1 c_0^2 + 2a_3 c_0 c_1$, $\gamma_3 = a_0 c_0^2 + a_1 c_0 c_1 + a_3 c_1^2$

Substitute $q = \dot{\Phi}_r$, yields $\ddot{\Phi}_r = \dot{q} = \frac{dq}{d\Phi_r} \dot{\Phi}_r = q \frac{dq}{d\Phi_r}$

The equation (34) becomes:

$$q dq = \left(a_2 \Phi_r - \frac{\gamma_1 t^2 + \gamma_2 t + \gamma_3}{\Phi_r^3} \right) d\Phi_r \quad (34)$$

This yields :

$$q^2 + q_0 = a_2 \Phi_r^2 + \frac{\gamma_1 t^2 + \gamma_2 t + \gamma_3}{\Phi_r^2} \quad (35)$$

with $q_0 = a_2 \Phi_r^2(t_0) + \frac{\gamma_3}{\Phi_r^2(t_0)} - \dot{\Phi}^2(t_0)$

In the case of an accelerate-motoring mode of the IM, both $\Phi_r(t)$ and $\dot{\Phi}_r(t)$ are positives, the equation (35) can be expressed as follows:

$$\dot{\Phi}_r = \sqrt{a_2 \Phi_r^2 + \frac{\gamma_1 t^2 + \gamma_2 t + \gamma_3}{\Phi_r^2} - q_0} \quad (36)$$

It's difficult to solve this first order differential equation, but it can be implemented in numerical form as an online solution of the minimum-energy rotor flux reference:

$$\Phi_r(k+1) = \Phi_r(k) + h \sqrt{a_2 (\Phi_r(k))^2 + \frac{\gamma_1 k^2 + \gamma_2 k + \gamma_3}{(\Phi_r(k))^2} - q_0} \quad (37)$$

3.2.3 Constant motor speed

Because of the proportionality of the load torque to the motor speed, a constant load torque corresponds to a constant motor speed. We can allow to the motor speed reference, the following expression:

$$\Omega^* = c_2 \quad (38)$$

The same development of the preceding paragraph from the first equation of the system (31) yields the following expression:

$$\dot{\Phi}_r = \pm \sqrt{a_2 \Phi_r^2 + \frac{\gamma_0}{\Phi_r^2} - q_1} \quad (39)$$

with $\gamma_0 = a_3 c_2^2$ and $q_1 = a_2 \Phi_r^2(t_0) + \frac{\gamma_0}{\Phi_r^2(t_0)} - \dot{\Phi}_r^2(t_0)$.

By imposing $\dot{\Phi}_r(t_0) = \sqrt{a_2} \Phi_r(t_0) - \frac{\sqrt{\gamma_0}}{\Phi_r(t_0)}$, the equation (39) becomes:

$$\dot{\Phi}_r = \pm \left| \sqrt{a_2} \Phi_r - \frac{\sqrt{\gamma_0}}{\Phi_r} \right| \quad (40)$$

For each case of $\Phi_r > \frac{1}{4} \sqrt{\frac{\gamma_0}{a_2}}$ or $\Phi_r < \frac{1}{4} \sqrt{\frac{\gamma_0}{a_2}}$, the real solution is given as follows:

$$\dot{\Phi}_r = -\sqrt{a_2} \Phi_r + \frac{\sqrt{\gamma_0}}{\Phi_r} \quad (41)$$

Substitute $s(t)$ for $\Phi_r(t)$, as follows: $s = (\Phi_r)^2$; a first-order differential equation can be obtained from (41):

$$\dot{s}(t) = -2\sqrt{a_2} s(t) + 2\sqrt{\gamma_0} \quad (42)$$

where $s(t_0) = (\Phi_r(t_0))^2$ and $s(T) = (\Phi_r(T))^2$.

From (43), $s(t)$ can be determined as follows:

$$s(t) = e^{-\int_{t_0}^t 2\sqrt{a_2} d\tau} \left(s(t_0) e^{\int_{t_0}^t 2\sqrt{a_2} d\tau} + \int_{t_0}^t 2\sqrt{\gamma_0} e^{\int_{t_0}^{\xi} 2\sqrt{a_2} d\tau} d\xi \right) \quad (43)$$

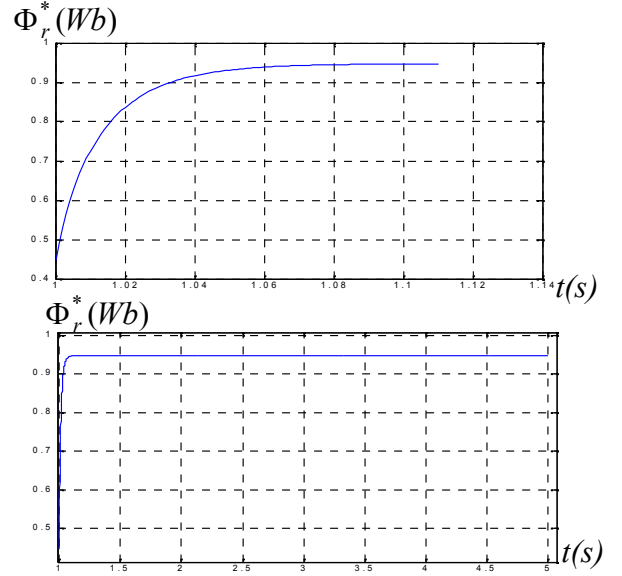
This yields the following solution:

$$s_1(t) = \left(\Phi_r^{*2}(t_0) e^{-2\sqrt{a_2}(t-t_0)} + \sqrt{\frac{\gamma_0}{a_2}} \left(1 - e^{-2\sqrt{a_2}(t-t_0)} \right) \right) \quad (44)$$

The optimal rotor flux is given as follows:

$$\Phi_r^*(t) = \sqrt{\Phi_r^{*2}(t_0) e^{-2\sqrt{a_2}(t-t_0)} + \sqrt{\frac{\gamma_0}{a_2}} \left(1 - e^{-2\sqrt{a_2}(t-t_0)} \right)} \quad (45)$$

Figures (1) and (2) illustrate the time-varying curve of the minimum-energy rotor flux at the following two conditions respectively:



Figs. 1 and 2: Minimum-energy rotor flux versus time

Figure (1): $t_0 = 1s$ and $T = 0.11s$

Figure (2): $t_0 = 1s$ and $T = 4s$

4. Deadbeat control of the rotor flux level

These kinds of problems need a wide range of the rotor flux magnitude variation. A deadbeat response have been chosen to regulate the rotor flux level [12] by means of a digital controller producing a flux level response with zero-state error and having a finite minimum settling time in its step response

In practice, the magnetic saturation has an important influence on rotor flux level control. The mutual inductance M varies widely in the motor operating region [12]. For the computation of optimal rotor flux, the mutual inductance values of the machine under this consideration should be used. If incorrect mutual inductance value is used in controller, it may cause instantaneous errors in rotor flux level. Thus it is essential to use the accurate mutual value in order to have a good dynamics in the transient change of the torque. Then in order to examine and describe the detuned effect of the magnetizing inductance, various models can be used. One of the most adequate representations of magnetizing rotor current i_{mr}

in terms of rotor flux Φ_r taking into account magnetic saturation consists on assigning a steady state law to the variation of the magnetizing rotor current. The following analytic polynomial function is chosen, as follows:

$$i_{mr} = c_1\Phi_r + c_2\Phi_r^3 + c_3\Phi_r^5 \quad (46)$$

The variation of the mutual in steady state operation can be given by:

$$M = \frac{\Phi_r}{i_{mr}} = \frac{1}{c_1 + c_2\Phi_r^2 + c_3\Phi_r^4} \quad (47)$$

with: c_1, c_2 and c_3 are the coefficients of the magnetizing curve.

In the case of the asynchronous machine, the coefficients: c_1, c_2 and c_3 are computable with a good precision by using the real curve $M = f(\Phi_r)$. The characteristic is accessible, by means of a no-load test to be realized on the machine. Figure 3 shows the results of the variation of M according to the rotor flux.

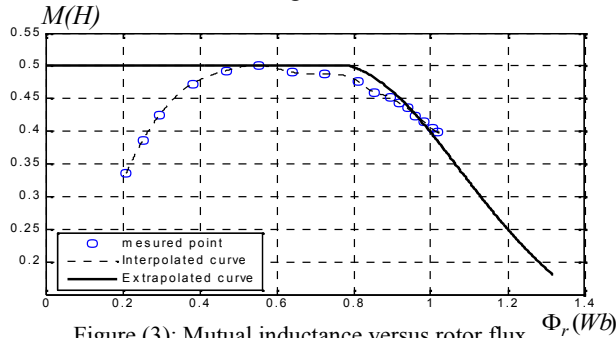


Figure (3): Mutual inductance versus rotor flux. Φ_r (Wb)

By using the least squares method, one can deduce the values of many constants:

$$c_1 = 2.6926, c_2 = -2.6528 \text{ et } c_3 = 2.4749.$$

In a rotation reference frame, the rotor flux can be expressed using the proposed control strategy as follows:

$$\Phi_r(s) = \frac{M}{1+sT_r} I_{sd} \quad (48)$$

The reference value of current I_{sd}^* can be written as follows [12]:

$$I_{sd}^*(nT_s) = I_{sd}^*((n-1)T_s) + \frac{1}{1 - \exp(-\frac{T_s}{T_r})} \left[\frac{\Phi_r^*(nT_s)}{M(nT_s)} - \frac{\hat{\Phi}_r(nT_s)}{M((n-1)T_s)} \right] - \frac{\exp(-\frac{T_s}{T_r})}{1 - \exp(-\frac{T_s}{T_r})} \left[\frac{\Phi_r^*((n-1)T_s)}{M((n-1)T_s)} - \frac{\hat{\Phi}_r((n-1)T_s)}{M((n-2)T_s)} \right] \quad (49)$$

where T_s is the settling time, T_r is the rotor time constant, $\hat{\Phi}_r(iT_s)$ and $M(iT_s)$ are respectively the estimated rotor flux and the mutual inductance at sampling time iT_s .

5. Optimal Control in Field Oriented Control (ORFOC)

The ORFOC of the IM is given in figure 4.

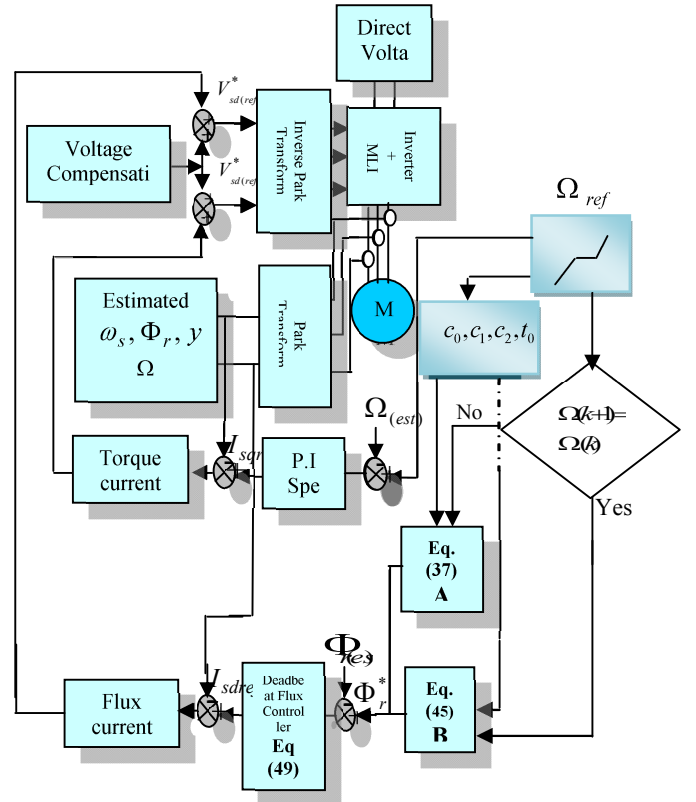


Figure (4): ORFOC scheme of the IM.

The RFOC drive of the IM, given in Figure 4, is initialized through applying a motor speed reference. Every step, the test-bloc decides on which type of operation mode the IM will undergo: steady state or transient mode (the step size was chosen enough in order to disable the undesired evaluation errors). One of the blocs A or B predicts the minimum-energy rotor flux reference. This reference will be implemented in the rotor flux closed-loop.

Finally, an optimal rotor flux current by means of the deadbeat controller is delivered to the remaining parts of RFOC drive. On the other hand, a transient torque current reference will be delivered to the rest of the RFOC drive.

The simulations results are carried out on a three-phase IM, 380V, 1.5KW, 50Hz and 4 poles, squirrel cage induction motor. The motor parameters are $R_r = 4.2$ ohm, $R_s = 6.06$ Ohm, $L_r = L_s = 0.462$ and the mechanical parameters are: $J_m = 0.0049$ and $K_t = 0.067$. For the conventional RFOC algorithm, the mutual inductance M is chosen constant and equal to 0.44H. But for the optimized RFOC using the minimum-energy rotor flux, M will be chosen as the expression in equation (48).

6. Simulation results

6.1. Comparison results with the conventional RFOC

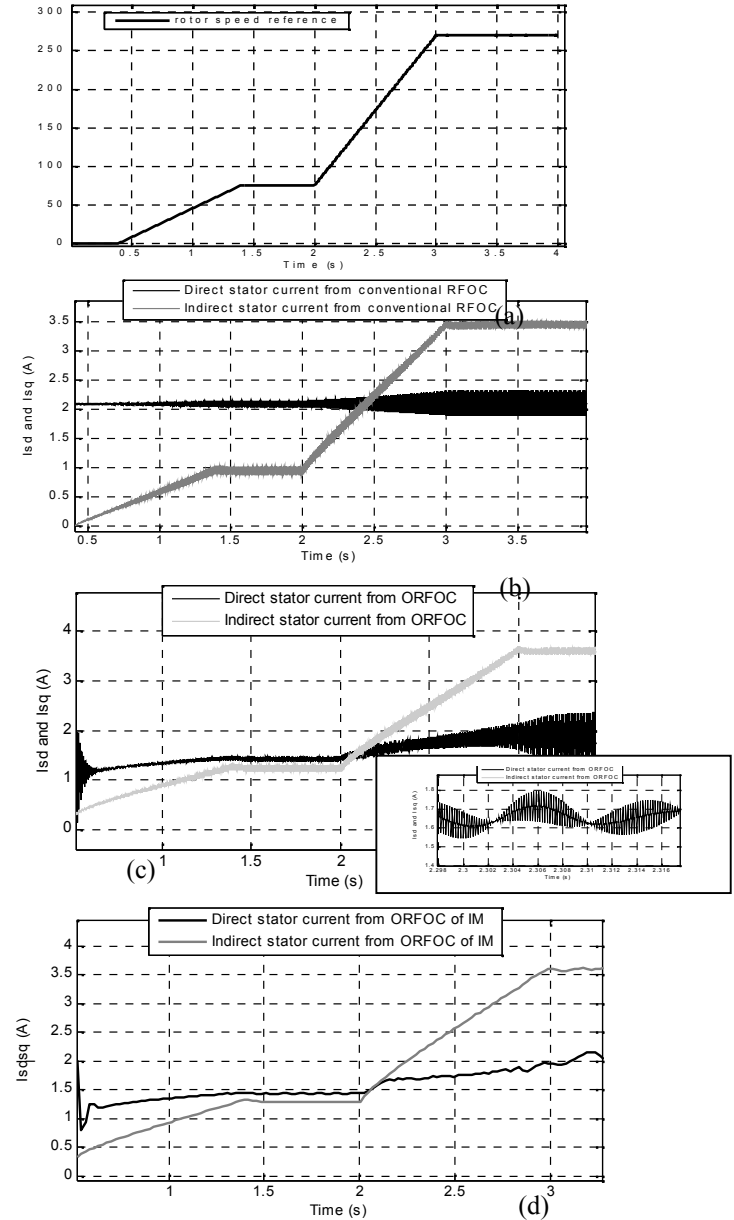
By applying a rotor speed reference as shown in Figure 5(a) a rotor flux reference is deduced from equations (37) or (45) when the IM is at either transient or steady state operation. Both of the rotor flux and the torque controllers deliver to the rest of the RFOC respectively the reference of optimal direct stator current and the reference of the desired indirect stator current reference.

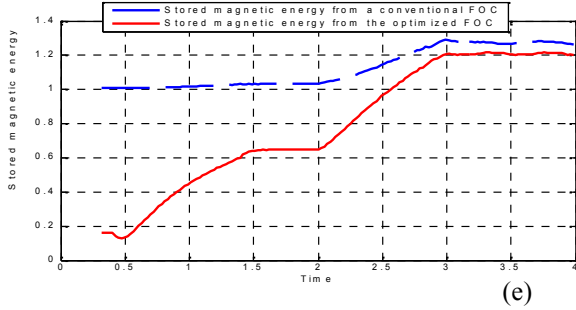
It is obvious to remark from Figure 5(b) that the flux current given from the conventional RFOC remains constant during the rotor speed increasing accompanied by an increasing torque current. Figure 5(c) shows that the flux current delivered by the ORFOC registers a significant decrease compared to the one delivered by the conventional RFOC. This means that the presented method saves energy.

In order to compare the different levels of stator currents, power factor, efficiency and energy delivered from different strategies of RFOC, and because they register high harmonic profiles, we have simplified these signals as shown in Figure (5-(c)) compared to the Figure (5-(d)): by picking up one point every $900T_s$ second with T_s is the settling period of the space vector modulation used in the RFOC. This period is chosen for this case equal to $\frac{1}{5000}$ (s). The accompanied Figure of 5-(c) is the zoomed version of the direct stator current in the interval [2.9s, 3.1s] of time.

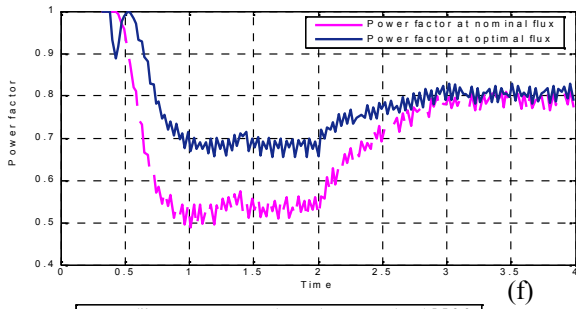
On the other hand, Figures 5(e) and 5(f) show the magnetic energy and the power factor, respectively. These results given by the proposed ORFOC compared to those delivered from the conventional RFOC prove that the minimization of the cost function performed by the proposed method causes a stored magnetic energy saving and consequently a power factor maximization. By comparing the energy consumption of the IM under ORFOC to a conventional one in Figure 5(g), Shows that

minimum energy is occurred in all the motor speed range as shown in the accompanied Figures for low, medium and high motor speed level.

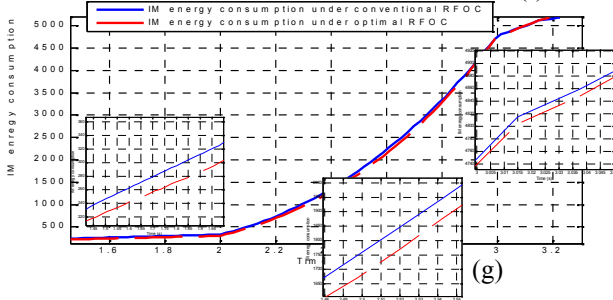




(e)



(f)



(g)

Figure (5): (a): Rotor speed reference; (b): Rotor flux and torque currents; (c) and (d): Real and simplified curves respectively of the rotor flux and torque currents; (e): Stored magnetic energy in the IM; (f): Power factor of the IM; (g): Total IM energy

6.2. Comparison results with other controller techniques

6.2.1 The sub-optimal energy method strategy [6], [14]

This minimum-energy method defined as a sub-optimal energy method (SOEM) was introduced by C. Canudas de Wit et al. [6], [14], and based on the theory of the optimal control. This method is subjected to the energy consumption optimization of the IM. By using a simplified IM energy model and defining a correspondent cost function, the presented optimal control problem showed a strategy of minimizing this cost function with dynamic constraint of the rotor flux.

From any load torque trajectory, an approximated optimal solution or sub-optimal solution was fairly determined referring to the low-frequency load torque trajectory and giving as follows [6]:

$$\Phi_r^* = \beta \sqrt{|y_d|}, \quad \beta = \left(\frac{b^2}{c^2 \left(a^2 + \frac{b^2 q}{r} \right)} \right)^{\frac{1}{4}} = \frac{\Phi_{rn}^*}{\sqrt{y_n}} \quad (50)$$

where Φ_{rn}^* and y_n are the nominal flux and nominal torque respectively β which is a function of the weighting factors.

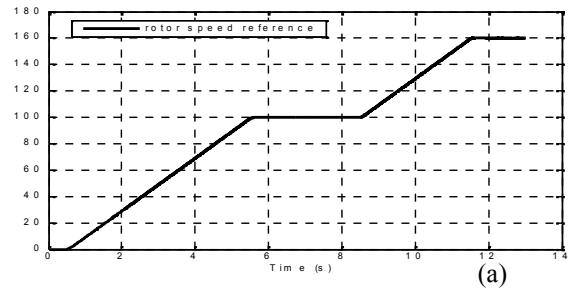
For a known time-variant trajectory $y_d(t)$, the authors supposed that the following approximate solution:

$$\hat{\Phi}_r^*(t) = \beta \sqrt{|y_d|} \quad (51)$$

uniformly approaches the optimal solution if the first and second time-derivative of the desired torque are small (which mean for the low-frequency of the load torque trajectory).

6.2.1.1 Comparison results

From a transient rotor speed reference showed in Figure 6(a), a comparative study in both of transient and steady state regimes was performed. These results are subjected to four different RFOC, giving respectively: the RFOC operating with SOEM solution, RFOC operating at rated rotor flux, RFOC operating with the proposed minimum-energy rotor flux Φ_r^* without magnetic effect and the RFOC operating also with Φ_r^* but by taking into account the saturation effect. From Figures 6(b) and 6(c), it is obvious to observe that the RFOC taking into account the magnetic effect and operating with Φ_r^* leads to the best IM energy consumption decreasing. Figures 6(d) and 6(c) confirm this result with high quality of the IM efficiency increasing obtained by the same type of RFOC for either light or medium rotor speed.



(a)

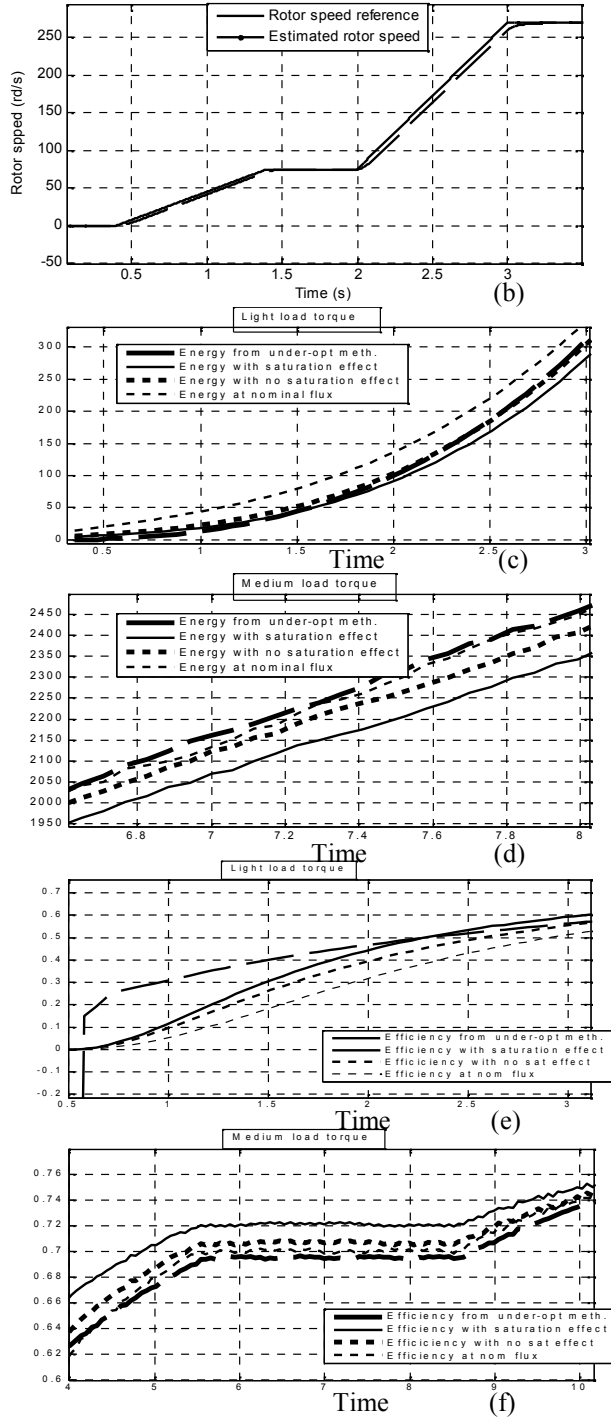


Figure (6): (a): Rotor speed reference; (b): Estimated and reference of rotor speed; (c): Total IM energy consumption, at light rotor speed range, (d): Total IM energy consumption, at medium rotor speed range, (e): IM Efficiency, at light rotor speed; (f): IM Efficiency, at medium rotor speed.

6.2.1 Minimum-Loss Control: (MLC) [16]

This proposed method was presented by Jae Ho Chang et al., section IV in [16]. In this study, a simple method is presented to determine the appropriate flux level considering copper loss in the steady state to minimize the energy consumption of the IM.

A field oriented control is considered with a steady state IM operation.

From minimizing the IM loss expression with respect to the rotor flux, the authors considered the following expression of the optimal rotor flux given by the equation (52):

$$\hat{\Phi}_r^*(t) = 2 \left(\frac{R_s L_r^2 + R_r M^2}{R_s} \right)^{\frac{1}{4}} \left(\frac{y_d}{3p} \right)^{\frac{1}{2}} \quad (52)$$

with y_d : as the desired torque.

The authors in this study required that this minimum-loss control is applied to get high efficiency during the steady state only.

6.2.2.1 Comparison results

To demonstrate the merits of the ORFOC using our proposed minimum-energy method, comparison results with MLC are performed. By implementing as a reference in the RFOC's flux controller a minimum-energy rotor flux trajectory giving from the equation (45) as well as the proposed optimal rotor flux [16], giving by the equation (52), and by keeping the same rotor speed reference trajectory giving by the Figure 6(a), simulation results of the consumed energy of the IM under these different RFOC are showed in Figure 7. It's obvious to observe the energy consumption decrease from our proposed ORFOC compared to the RFOC's results using the MLC strategy.

This decrease is fairly registered also in the steady state operation.

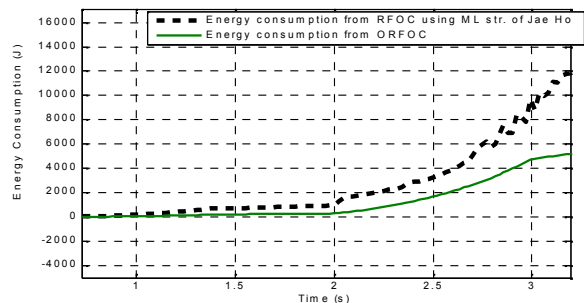


Figure (7): Energy consumption evolution from both of the proposed ORFOC and the RFOC using MLC strategy.

7. Conclusion

In this paper, a minimum-energy approach is presented. Based on the optimal control theory, this approach provides a cost function given as weighted sum of an IM energy-power model. In order to obtain a minimum-energy rotor flux trajectory, the presented task is to minimize this cost function constrained to a two dynamic equations of the rotor flux and the motor speed. By applying the Euler-Lagrange resolution, analytic solution is given in the case of steady state regime of the load and a recursive formula of the optimal rotor flux is given in transient regime and especially for an accelerated IM. Those minimum-energy rotor flux norms are implemented in the RFOC for a given operating mode. This control law develops the magnetic saturation effect on the motor parameters and a deadbeat control instead of the Proportional Integral in the rotor flux closed-loop is provided.

A comparative study is given and proves the validity of the proposed minimum-energy approach. This optimal rotor flux oriented control is compared to the one using the SOEM solution which is experimentally confirmed. The Comparison study is made also with the results given by a conventional RFOC and with an ORFOC but without taking into account the saturation effect on the motor parameters.

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