

MINIMUM-ENERGY CONSUMPTION OF A DOUBLE STATOR INDUCTION MOTOR OPERATING IN DYNAMIC REGIME

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Abstract: In this paper a problem of energy optimization of a double stator induction motor (DSIM) is considered. Using the concept of a rotor field oriented control (RFOC); the DSIM Blondel-Park model is used as dynamic constraints in an optimal control problem. A cost function consists of linear combination of magnetic energy, copper losses and mechanical power is minimized in order to find minimum-energy rotor flux trajectories. From calculus of variations a system of nonlinear differential equations are obtained, and analytical solutions are achieved in the case of accelerate and decelerate mode of the DSIM speed. These solutions gave during the two modes a time varying trajectories of a minimum-energy rotor flux. These trajectories are implemented in the optimal rotor flux oriented control (ORFOC) of the DSIM and compared to the conventional RFOC at different dynamic regime of the DSIM. Simulation results are given and improved the effectiveness of the proposed strategy.

Keywords: double-star induction machine; field-oriented control; optimal control, energy minimization; dynamic regime.

1. Introduction

The theme of current research in the control of (DSIM) operating at variable speed is the energy optimization control. The induction motor is less efficient in terms of energy than the synchronous motor. It is that this machine requires stator currents continuously even in null torque to maintain the rotor flux. This increases substantially losses in rotor and stator. In this context, the literature like [7], [10], [13] and [14] addresses this problem in two aspects:

- maximization of the torque

- maximization of the efficiency

We are interested in this study to the second aspect because it is the most considered in embedded systems [8] and [11].

Despite significant progress, we still seem to exist significant opportunities for improving the cost function, by intervening in their operating principles particularly at variable speed and their control laws. This is so, we are interested in extending a hand to the principle of minimizing energy consumption of the DSIM and secondly to use the rotor flux instead of the iron flux [15].

In this paper a strategy of optimizing energy of the DSIM in transitory regime using calculus of variation theory is developed. An integral function is considered and decomposed into a weighted sum of power-energy of the DSIM for a given time interval. This function called the cost function [R.A] will be constrained to boundaries conditions and to rotor flux and motor speed dynamical equations which are developed from the DSIM transient model in a turning (d,q) reference frame [6]

Using the Euler-Lagrange to resolve the proposed Optimal Control Problem (OCP), an optimal rotor flux is giving. This solution provides the lowest IM's energy consumption along the given motor speed range. To solve this problem, an important mathematical background is needed.

2. Nomenclature

R_s : Stator resistances,

R_r : Rotor resistances,

L_s : Stator inductance,

L_{sp} :Principal cyclic stator inductance,
 L_r :Rotor inductance,
 M_{sr} : The mutuel inductance,
 p : the poles number
 V_{s1}, V_{s2} :Respectively the voltage of stator 1 and stator2,
 I_{s1}, I_{s2} :Respectively the current of stator 1 and stator2,
 ϕ_{s1}, ϕ_{s2} :Respectively the flux of stator 1 and stator2,
 V_r : The rotor voltage,
 I_r : The rotor current,
 ϕ_r :The rotor flux,
 C_{em} :The electromagnetic torque,
 C_r :The load torque,
 k_t : The load torque constant,
 J_m :The moment of inertia,
 g : The slip;
 ω_r :The rotor speed,
 ω_s :The stator frequency,
 ω_g : The slip speed,
 Ω : mechanical speed

3. Modeling of the DSIM

From the end of the 1920s, the machines with two three-phase windings in the stator were introduced to increase the power of synchronous generators. The DSIM requires a double three-phase supply and has many advantages. Not only does give reduced torque oscillations, but it also requires less powerful electronic components as the current flowing in a six-phase machine is less than that flowing in a three-phase machine. However, as the use of an inverter is necessary when feeding a double star induction machine, this may result in supplementary losses, since such an inverter is a harmonic generator.

The double-star induction motor consists of a standard simple squirrel-cage rotor and two

separate three-phase stator windings. The windings of the DSIM are shown in Fig.1.

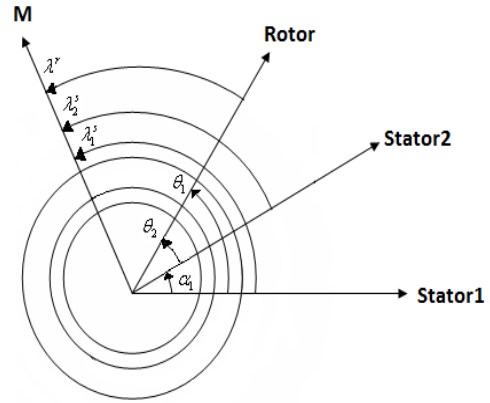


Figure 1:Representation of the winding of the MADE

The mathematical model of the machine in the natural frame is written as a set of state equations, as follows:

$$[V_{s1}] = [R_s][I_{s1}] + \frac{d[\phi_{s1}]}{dt} \quad (1)$$

$$[V_{s2}] = [R_s][I_{s2}] + \frac{d[\phi_{s2}]}{dt} \quad (2)$$

$$[V_r] = [R_r][I_r] + \frac{d[\phi_r]}{dt} \quad (3)$$

With :

$$[V_{s1}] = [V_{s1a}] \quad [V_{s1b}] \quad [V_{s1c}]^t$$

$$[V_{s2}] = [V_{s2a}] \quad [V_{s2b}] \quad [V_{s2c}]^t$$

$$[V_r] = [V_{ra}] \quad [V_{rb}] \quad [V_{rc}]^t$$

$$[I_{s1}] = [I_{s1a}] \quad [I_{s1b}] \quad [I_{s1c}]^t$$

$$[I_{s2}] = [I_{s2a}] \quad [I_{s2b}] \quad [I_{s2c}]^t$$

$$[I_r] = [I_{ra}] \quad [I_{rb}] \quad [I_{rc}]^t$$

$$[\phi_{s1}] = [\phi_{s1a}] \quad [\phi_{s1b}] \quad [\phi_{s1c}]^t$$

$$[\phi_{s2}] = [\phi_{s2a}] \quad [\phi_{s2b}] \quad [\phi_{s2c}]^t$$

$$[\phi_r] = [\phi_{ra}] \quad [\phi_{rb}] \quad [\phi_{rc}]^t$$

$$[R_s] = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}; [R_r] = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix}$$

The stator and rotor flux are expressed by equations as follows:

$$[\varphi_{s1}] = [L_s][I_{s1}] + [M_{s12}][I_{s2}] + [M_{s1r}][I_r] \quad (4)$$

$$[\varphi_{s2}] = [L_s][I_{s2}] + [M_{s21}][I_{s1}] + [M_{s2r}][I_r] \quad (5)$$

$$[\varphi_r] = [L_r][I_r] + [M_{sr}](I_{s1} + I_{s2}) \quad (6)$$

With:

$$[L_s] = \begin{bmatrix} L_s + L_{ms} & -L_{ms}/2 & -L_{ms}/2 \\ -L_{ms}/2 & L_s + L_{ms} & -L_{ms}/2 \\ -L_{ms}/2 & -L_{ms}/2 & L_s + L_{ms} \end{bmatrix}$$

$$[L_r] = \begin{bmatrix} L_r + L_{mr} & -L_{ms}/2 & -L_{ms}/2 \\ -L_{ms}/2 & L_r + L_{mr} & -L_{ms}/2 \\ -L_{ms}/2 & -L_{ms}/2 & L_r + L_{mr} \end{bmatrix}$$

$$[M_{s1r}] = M_{sr} \begin{bmatrix} \cos(\theta_1) & \cos(\theta_1 + 2\pi/3) & \cos(\theta_1 + 4\pi/3) \\ \cos(\theta_1 + 4\pi/3) & \cos(\theta_1) & \cos(\theta_1 + 2\pi/3) \\ \cos(\theta_1 + 2\pi/3) & \cos(\theta_1 + 4\pi/3) & \cos(\theta_1) \end{bmatrix}$$

$$[M_{s21}] = [M_{s12}] = L_{ms} \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_1 + 2\pi/3) & \cos(\alpha_1 + 4\pi/3) \\ \cos(\alpha_1 + 4\pi/3) & \cos(\alpha_1) & \cos(\alpha_1 + 2\pi/3) \\ \cos(\alpha_1 + 2\pi/3) & \cos(\alpha_1 + 4\pi/3) & \cos(\alpha_1) \end{bmatrix}$$

$$[M_{s2r}] = M_{sr} \begin{bmatrix} \cos(\theta_2) & \cos(\theta_2 + 2\pi/3) & \cos(\theta_2 + 4\pi/3) \\ \cos(\theta_2 + 4\pi/3) & \cos(\theta_2) & \cos(\theta_2 + 2\pi/3) \\ \cos(\theta_2 + 2\pi/3) & \cos(\theta_2 + 4\pi/3) & \cos(\theta_2) \end{bmatrix}$$

M_{sr} : the maximum value of the coefficient of the mutual inductance between the stator and the rotor.

L_{mr} : the maximum value of the coefficients of the mutual inductance of the rotor.

L_{ms} : the maximum value of the coefficients of the mutual inductance of the stator.

The mechanical equation is given by:

$$J_m \frac{d\Omega}{dt} = C_{em} - C_r \quad (7)$$

Where:

C_r is chosen proportional to the motor speed.

3.1. Park model of the DSIM

Park model is based on a transformation of three-phase system of axes (a, b, c) to an equivalent two-phase system of axes (d, q).

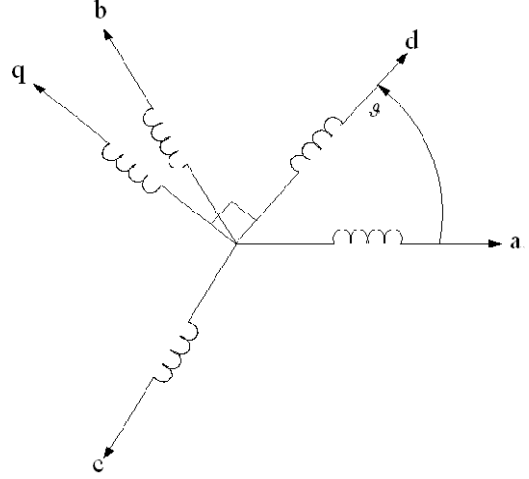


Figure 2: Park transformation

In this work, we use the reference frame linked to the rotating field, for the modeling and the control of the DSIM. In this case, the model of the DSIM becomes:

$$V_{sd1} = R_s i_{sd1} + \frac{d}{dt} \varphi_{sd1} - \omega_s \varphi_{sq1} \quad (8)$$

$$V_{sq1} = R_s i_{sq1} + \frac{d}{dt} \varphi_{sq1} + \omega_s \varphi_{sd1} \quad (9)$$

$$V_{sd2} = R_s i_{sd2} + \frac{d}{dt} \varphi_{sd2} - \omega_s \varphi_{sq2} \quad (10)$$

$$V_{sq2} = R_s i_{sq2} + \frac{d}{dt} \varphi_{sq2} + \omega_s \varphi_{sd2} \quad (11)$$

$$0 = R_r i_{rd} + \frac{d}{dt} \varphi_{rd} - \omega_g \varphi_{rq} \quad (12)$$

$$0 = R_r i_{rq} + \frac{d}{dt} \varphi_{rq} + \omega_g \varphi_{rd} \quad (13)$$

By applying the transformation of Park to the flux's equations, we obtain:

$$\varphi_{sd1} = L_s i_{sd1} + \frac{3}{2} L_{ms} i_{sd1} + \frac{3}{2} L_{ms} i_{sd2} + \frac{3}{2} M_{sr} i_{rd} \quad (14)$$

$$\varphi_{sq1} = L_s i_{sq1} + \frac{3}{2} L_{ms} i_{sq1} + \frac{3}{2} L_{ms} i_{sq2} + \frac{3}{2} M_{sr} i_{rq} \quad (15)$$

$$\varphi_{sd2} = L_s i_{sd2} + \frac{3}{2} L_{ms} i_{sd2} + \frac{3}{2} L_{ms} i_{sd1} + \frac{3}{2} M_{sr} i_{rd} \quad (16)$$

$$\varphi_{sq2} = L_s i_{sq2} + \frac{3}{2} L_{ms} i_{sq2} + \frac{3}{2} L_{ms} i_{sq1} + \frac{3}{2} M_{sr} i_{rq} \quad (17)$$

$$\varphi_{rd} = L_r i_{rd} + \frac{3}{2} M_{sr} i_{sd1} + \frac{3}{2} M_{sr} i_{sd2} \quad (18)$$

$$\varphi_{rq} = L_r i_{rq} + \frac{3}{2} M_{sr} i_{sq1} + \frac{3}{2} M_{sr} i_{sq2} \quad (19)$$

The electromagnetic torque is given by the following equation:

$$C_{em} = p(\varphi_{sd1} i_{sq1} + \varphi_{sd2} i_{sq2} - \varphi_{sq1} i_{sd1} + \varphi_{sq2} i_{sd2}) \quad (20)$$

It can be described also as:

$$C_{em} = \frac{3}{2} M_{sr} p \left(\varphi_{rd} (i_{sq1} + i_{sq2}) + \varphi_{rq} (i_{sd1} + i_{sd2}) \right) \quad (21)$$

4. The principle of field-oriented control (FOC) approach to the DSIM

Field Oriented Control represents the method by which the rotor flux is considered as a basis for creating a reference frame for one of the other fluxes (stator or air gap) with the purpose of decoupling the torque and flux producing components of the stator current. The best way is to confuse the vector space with its direct component as follow: $\varphi_r = \varphi_{rd}$ and $\varphi_{rq} = 0$.

The model of the DSIM in the (d, q) reference becomes then:

$$V_{sd1} = R_s i_{sd1} + \frac{d}{dt} \varphi_{sd1} - \omega_s \varphi_{sq1} \quad (22)$$

$$V_{sq1} = R_s i_{sq1} + \frac{d}{dt} \varphi_{sq1} + \omega_s \varphi_{sd1} \quad (23)$$

$$V_{sd2} = R_s i_{sd2} + \frac{d}{dt} \varphi_{sd2} - \omega_s \varphi_{sq2} \quad (24)$$

$$V_{sq2} = R_s i_{sq2} + \frac{d}{dt} \varphi_{sq2} + \omega_s \varphi_{sd2} \quad (25)$$

$$0 = R_r i_{rd} + \frac{d}{dt} \varphi_r \quad (26)$$

$$0 = R_r i_{rq} + \omega_g \varphi_{rd} \quad (27)$$

With the same way, we define the expressions of flux:

$$\varphi_{sd1} = L_s i_{sd1} + \frac{3}{2} L_{ms} i_{sd1} + \frac{3}{2} L_{ms} i_{sd2} \quad (28)$$

$$\varphi_{sq1} = L_s i_{sq1} + \frac{3}{2} L_{ms} i_{sq1} + \frac{3}{2} L_{ms} i_{sq2} + \frac{3}{2} M_{sr} i_{rq} \quad (29)$$

$$\varphi_{sd2} = L_s i_{sd2} + \frac{3}{2} L_{ms} i_{sd2} + \frac{3}{2} L_{ms} i_{sd1} \quad (30)$$

$$\varphi_{sq2} = L_s i_{sq2} + \frac{3}{2} L_{ms} i_{sq2} + \frac{3}{2} L_{ms} i_{sq1} + \frac{3}{2} M_{sr} i_{rq} \quad (31)$$

$$\varphi_{rd} = \frac{3}{2} M_{sr} i_{sd1} + \frac{3}{2} M_{sr} i_{sd2} \quad (32)$$

$$0 = L_r i_{rq} + \frac{3}{2} M_{sr} i_{sq1} + \frac{3}{2} M_{sr} i_{sq2} \quad (33)$$

The electromagnetic torque can be written as follows:

$$C_{em} = \frac{3}{2} M_{sr} p \left(\varphi_{rd} (i_{sq1} + i_{sq2}) \right) \quad (34)$$

5. Full-order dynamic model of DSIM

The full-order model of the DSIM viewed from the synchronous rotating reference frame is given by the following system [1]:

$$\begin{cases} \dot{I}_{s(d,q)} = -(\gamma I + (\dot{\rho} + p\dot{\Omega})J)I_{s(d,q)} & (35) \\ \dot{\varphi}_{r(d,q)} = -(aI + \dot{\rho}J)\varphi_{r(d,q)} + bI & (36) \end{cases}$$

$$\begin{cases} \dot{\Omega} = -\frac{K_l}{J_m} + \frac{y}{J_m} & (37) \end{cases}$$

Where:

$$I_{s(d,q)} = \begin{pmatrix} I_{s1d} + I_{s2d} \\ I_{s1q} + I_{s2q} \end{pmatrix} = \begin{pmatrix} I_{sd} \\ I_{sq} \end{pmatrix}$$

$$\sigma_2 = 1 - \left(\frac{M^2}{L_r L_{sp}} \right); \varphi_{r(d,q)} = \begin{pmatrix} \varphi_{rd} \\ \varphi_{rq} \end{pmatrix}$$

$$\sigma_1 = 1 - \left(\frac{M^2}{L_r L_s} \right); a = \frac{R_r}{L_r}; b = aM$$

$$\gamma = \frac{1}{\sigma_1 L_s + \sigma_2 L_{sp}} \left(R_s + \frac{M^2}{L_r^2} R_r \right)$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$

$$V_{s(d,q)} = \begin{pmatrix} V_{s1d} + V_{s2d} \\ V_{s1q} + V_{s2q} \end{pmatrix} = \begin{pmatrix} V_{sd} \\ V_{sq} \end{pmatrix}.$$

5.1. Reduced model of the DSIM

The stator current is taking as an input control of the system. A high gain control current loop is chosen in order to simplify the optimization algorithm efficiency [1]. Such choice permits to apply a reduced order current fed DSIM model; the current loop is given as: [6]

$$V_{s(d,q)} = \frac{\sigma_1 L_s + \sigma_2 L_{sp}}{\varepsilon} (U - I_{s(d,q)}) \quad (38)$$

with:

$$U = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} I_{s1d} + I_{s2d} \\ I_{s1q} + I_{s2q} \end{pmatrix}$$

Which $0 < \varepsilon < 1$ and U is a new command of the system. By allowing for this control, the reduced model can be obtained via the "singular perturbation theorem" [15]. This involves from (35), (36), (37) and from (2.1), a reduced model of the DSIM is built as follows:

$$\begin{cases} \dot{\varphi}_{r(d,q)} = -(aI + \rho I) \varphi_{r(d,q)} + bU & (39) \end{cases}$$

$$\begin{cases} \dot{\Omega} = -\frac{K_1}{J_m} + \frac{cU^t J \varphi_{r(d,q)}}{J_m} & (40) \end{cases}$$

6. Energy model of DSIM

The instantaneous active power in the (d-q) rotating frame is given by:

$$P_a = 3/2 (V_{s(d,q)})^t I_{s(d,q)} \quad (41)$$

From equation (35), (36) and (37) the input power is given by:

$$\begin{aligned} P_a = & 3/2 (\sigma_1 L_s + \sigma_2 L_{sp}) \cdot \left((I_{s(d,q)})^t (I_{s(d,q)}) \right) + \\ & \gamma \cdot \left((I_{s(d,q)})^t (I_{s(d,q)}) \right) - \\ & 2\eta\alpha \left((\varphi_{r(d,q)})^t (I_{s(d,q)}) \right) + \eta p \Omega (\varphi_{r(d,q)})^t I_{s(d,q)} \end{aligned} \quad (42)$$

The relation between the stator and rotor current can be given as follows:

$$I_{r(d,q)} = \frac{1}{L_r} (\varphi_{r(d,q)} - L_s I_{s(d,q)}) \quad (43)$$

The instantaneous active power becomes then:

$$\begin{aligned} P_a = & 3/2 (\sigma_1 L_s + \sigma_2 L_{sp}) \cdot \left((I_{s(d,q)})^t (I_{s(d,q)}) \right) + \\ & \frac{1}{L_r} \cdot \left((\dot{\varphi}_{r(d,q)})^t (\varphi_{r(d,q)}) \right) - \\ & 3/2 \left[\left(R_s (I_{s(d,q)})^t (I_{s(d,q)}) \right) + \right. \\ & \left. \left(R_r (I_{r(d,q)})^t (I_{r(d,q)}) \right) \right] + \Omega Y \end{aligned} \quad (44)$$

Finally, the instantaneous active power is given as:

$$P_a = \frac{\partial}{\partial t} W + P_j + P_m \quad (45)$$

By means of a field-oriented control drive, we define from equation (44):

- the derivate of stored magnetic given as follows:

$$\frac{\partial}{\partial t} W = 3/2 \left(\left(\frac{\sigma_1 L_s + \sigma_2 L_{sp}}{2} \right) \cdot (u_1^2 + u_2^2) + \frac{1}{2L_r} \varphi_r^2 \right) \quad (46)$$

- the Joule losses:

$$\begin{aligned} P_j = & 3/2 \left[\left(R_s (I_{s(d,q)})^t (I_{s(d,q)}) \right) \left(R_r (I_{r(d,q)})^t (I_{r(d,q)}) \right) \right] \end{aligned} \quad (47)$$

By using equation (35) and (43), those losses can be expressed with respect to U and φ_r as follows:

$P_j =$

$$3/2 \left(\left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) \cdot (u_1^2 + u_2^2) - \frac{3 R_r}{2 L_r^2} \varphi_r^2 - \frac{3}{2 L_r} \frac{d\varphi_r^2}{dt} \right) \quad (48)$$

- The mechanical power of the DSIM:

$$P_m = \Omega Y \quad (49)$$

In term of rotor variables and torque current (I_{sq}), we get:

$$P_m = 3/2 \frac{M}{L_r} \varphi_r I_{sq} \Omega \quad (50)$$

7. The optimal problem control

An optimal control consists on minimization of function. In this case, the cost function can be defined as the integral of an index $f(I_{sd}; I_{sq}; \varphi_r; \Omega)$ given as follows:

$$J = \int_0^T f(I_{sd}; I_{sq}; \varphi_r; \Omega) dt \quad (51)$$

The index corresponds to the weighted sum:

$$f(I_{sd}; I_{sq}; \varphi_r; \Omega) = \varphi_1 W_L + \varphi_2 P_j + \varphi_3 P_m \quad (52)$$

The weighting factors ($\varphi_1; \varphi_2; \varphi_3$) are used to scale power-energy combined convex criteria terms defined above. Minimize the cost function provides two important advantages: first, minimizing the corresponding magnetic energy stored, allowing thus maximizing the power factor, the second being the minimization of losses in the winding thus increasing the machine efficiency.

Using equations (44), (46), (48) and (50), the cost function is given as follows:

$$J = 3/2 \int_0^T \left[\varphi_1 \left((\sigma_1 L_s + \sigma_2 L_{sp}) \cdot (u_1^2 + u_2^2) + \frac{1}{2 L_r} \cdot \varphi_r^2 \right) + \varphi_2 \left(\left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) (u_1^2 + u_2^2) - \frac{R_r}{L_r^2} \varphi_r^2 \right) - \frac{\varphi_2}{L_r} \frac{d}{dt} \varphi_r^2 + \varphi_3 \left(p \frac{M}{L_r} \varphi_r u_2 \Omega \right) \right] dt \quad (53)$$

The integral

$$\int_0^T \left(-\frac{1}{L_r} \frac{d}{dt} (\varphi_r^2) \right) dt = \frac{1}{L_r} (\varphi_r(0) - \varphi_r(T)) \quad (54)$$

, has no effect in the optimizing problem and can be omitted it from the integral. Considering the new control vector $U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, the system described by (39) and (40) can be defined as follows:

$$\dot{\varphi}_r = a \varphi_r + b u_1 \quad (55)$$

$$\dot{\Omega} = -\frac{K_1}{J_m} \Omega + \frac{c u_2 \varphi_r}{J_m} \quad (56)$$

Using the new constraint given by (55) and (56) the cost function given by (53), an optimal control problem can be presented as follows:

- Find the optimal control variables (u_1^* and u_2^*) that minimize the follows cost function:

$$J_r = \int_0^T (r_1 u_1^2 + r_2 u_2^2 + q_1 \varphi_r^2 + q_2 \varphi_r u_2 \Omega) dt \quad (57)$$

While respecting the dynamic constraints given in the system (55) and (56).

With:

$$r_1 = 3/4 (\sigma_1 L_s + \sigma_2 L_{sp}) \varphi_1 + 3/2 \left(R_s + R_r \left(\frac{M}{L_r} \right)^2 \right) \varphi_2$$

$$r_2 = r_1 + r_0 \alpha_3; q_1 = 3/4 \frac{\alpha_1}{L_r} - 3/2 \frac{R_r}{L_r^2} \alpha_2 + \alpha_3 q_0$$

$$q_2 = \frac{3 M}{2 L_r} \alpha_4$$

where the weighting factors r_1, r_2, q_1 and q_2 must be positives.

Otherwise, the task is to find an admissible control trajectory $U^* = \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix}$ generating the corresponding state trajectory φ_r^* defined as the optimal rotor flux to provide minimum of the cost function presented in equation (57).

The optimal control problem amounts to determining the optimized values of u_1^* and u_2^* which are defined as follows:

$$\begin{cases} u_1^* = \frac{1}{b}(\dot{\varphi}_r^* - a\varphi_r^*) \\ u_2^* = -\frac{J_m}{c\varphi_r} \dot{\Omega}^* + \frac{k_1}{J_m} \Omega^* \end{cases} \quad (58)$$

By injecting the expressions of u_1^* and u_2^* from the system (58) in the cost function in (57) we get:

$$J_r = \int_0^T \left(\gamma_1 \left(\frac{1}{b} (\dot{\varphi}_r^* - a\varphi_r^*) \right)^2 + \gamma_2 \left(\frac{J_m}{c\varphi_r} (\dot{\Omega}^* + \frac{k_1}{J_m} \Omega^*) \right)^2 + q_1 \varphi_r^2 + q_2 \varphi_r \left(\frac{J_m}{c\varphi_r} (\dot{\Omega} + \frac{k_1}{J_m} \Omega) \right) \right) \Omega dt \quad (59)$$

This yield:

$$\int_0^T J_r = \lambda_1 \dot{\varphi}_r^2 + \lambda_2 \frac{\Omega^2}{\varphi_r^2} + \lambda_3 \dot{\varphi}_r \varphi_r + \left(\frac{\lambda_4}{\varphi_r^2} + \lambda_5 \right) \Omega \dot{\Omega} + \lambda_6 \varphi_r^2 + \left(\frac{\lambda_7}{\varphi_r^2} + \lambda_8 \right) \Omega^2 dt \quad (60)$$

with

$$\lambda_1 = \frac{r_1}{a^2 M^2}; \lambda_2 = \frac{r_2 J_m^2}{c^2}; \lambda_3 = \frac{2r_2}{a M^2}; \lambda_4 = \frac{2r_2 J_m k}{c^2}; \lambda_5 = \frac{q_2}{c} J_m; \lambda_6 = \frac{r_1}{M^2} + q_1; \lambda_7 = \frac{r_2}{c^2} k_1^2; \lambda_8 = \frac{q_2}{c} k_1$$

$\lambda_{i=12 \text{ to } 8}$ must be positives constants defined accordingly to the condition on the constants $r_1; r_2; q_1$ and q_2 .

7.1. Euler-Lagrange Equation

This integral given by (60) can be expressed as follows:

$$J_r = \int_0^T L(\dot{\varphi}_r; \varphi_r; \dot{\Omega}; \Omega) dt \quad (61)$$

and can be solved using Euler-Lagrange equation with respect the follows condition:

- This integral has an absolute minimum value φ_r^* and Ω^* if their trajectory satisfies the following conditions:

$$\frac{\partial}{\partial \varphi_r} (L(\dot{\varphi}_r; \varphi_r; \dot{\Omega}; \Omega)) - \frac{\partial}{\partial t} \frac{\partial}{\partial \dot{\varphi}_r} (L(\dot{\varphi}_r; \varphi_r; \dot{\Omega}; \Omega)) = 0 \quad (62)$$

and

$$\frac{\partial}{\partial \Omega} (L(\dot{\varphi}_r; \varphi_r; \dot{\Omega}; \Omega)) - \frac{\partial}{\partial t} \left(\frac{\partial}{\partial \dot{\Omega}} (L(\dot{\varphi}_r; \varphi_r; \dot{\Omega}; \Omega)) \right) = 0 \quad (63)$$

Aiming to solve the equations (62) and (63) and by using the expression of the cost function in (60) the previous condition becomes:

$$\ddot{\varphi}_r = -a_0 \frac{\Omega^2}{\varphi_r^3} - a_1 \frac{\Omega}{\varphi_r^3} \dot{\Omega} + a_2 \varphi_r - a_3 \frac{\Omega^2}{\varphi_r^3} \quad (64)$$

$$\ddot{\Omega} = \frac{2}{\varphi_r} \dot{\varphi}_r \dot{\Omega} + a_4 \frac{\Omega}{\varphi_r} \dot{\varphi}_r + a_5 \Omega + a_6 \varphi_r^2 \Omega \quad (65)$$

were

$$a_0 = \frac{\lambda_2}{\lambda_1}; a_1 = \frac{\lambda_4}{\lambda_1}; a_2 = \frac{\lambda_6}{\lambda_1}; a_3 = \frac{\lambda_7}{\lambda_1};$$

$$a_4 = \frac{\lambda_4}{\lambda_2}; a_5 = \frac{\lambda_7}{\lambda_2}; a_6 = \frac{\lambda_8}{\lambda_2}$$

7.2. Linear Time-varying motor speed

In order to obtain an accelerate mode (transient regime), we have proposed a linear law for the motor speed as follows:

$$\Omega^* = c_0 t + c_1 \quad (66)$$

with $c_0 > 0$

Hence the second equation (65) has no physical meaning and can be skipped. By substituting the expression of Ω by Ω^* into the equation (64) we get:

$$\ddot{\varphi}_r \varphi_r^3 - a_2 \varphi_r^4 = -(\gamma_1 t^2 + \gamma_2 t + \gamma_3) \quad (67)$$

where:

$$\gamma_1 = a_3 c_0^2; \gamma_2 = a_1 c_0^2 + 2a_3 c_0 c_1$$

$$\gamma_3 = a_0 c_0^2 + a_1 c_0 c_1 + 2 + a_3 c_1^2$$

Despite the extreme difficulty to solve this equation (67), we tried to finding a solution using a mathematical calculation; so, we applied the method of integration by parts to this equation, during a time interval $[0, t]$.

Finally the optimal solution of the rotor flux is given by the following expression:

$$\varphi_r(t) = \sqrt[6]{\frac{30}{a_2} \left(\frac{a_2 \varphi_r^6(0)}{30} - \dot{\varphi}_r(0) \varphi_r^3(0) + \varphi_r^3(0)t + \frac{1}{5} \varphi_r^5(0) + \left(\frac{\gamma_1 t^4}{12} + \frac{\gamma_2 t^3}{6} + \frac{\gamma_3 t^2}{2} \right) - kt \right)} \quad (68)$$

Figures (1) illustrate the time-varying curve of the minimum-energy rotor flux for a speed ramp.

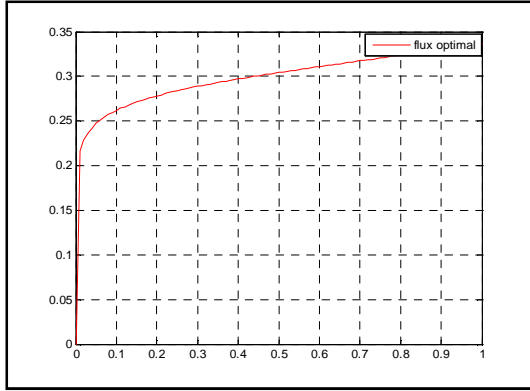


Fig3: Minimum energy rotor flux versus time

8. Deadbeat control of the rotor flux level

These kinds of problems need wide range of the rotor flux magnitude variation. A deadbeat response has been chosen to regulate the rotor flux level [12]. In this part, the mutual inductance value of the machine under this consideration is not used. In a rotation reference frame, the rotor flux can be

expressed using the proposed control approach as follows:

$$\varphi_r(s) = \frac{M}{1+sT_r} I_{sd} \quad (69)$$

The reference value of current I_{sd}^* can be written as follows:

$$I_{sd}^*(nT_s) = (I_{sd}^*(n-1)T_s) + \frac{1}{1 - \exp\left(-\frac{T_s}{T_r}\right)} \left[\frac{\varphi_r^*(nT_s)}{M(nT_s)} - \frac{\widehat{\varphi}_r(nT_s)}{M((n-1)T_s)} \right] - \frac{\exp\left(-\frac{T_s}{T_r}\right)}{1 - \exp\left(-\frac{T_s}{T_r}\right)} \left[\frac{\varphi_r^*((n-1)T_s)}{M((n-1)T_s)} - \frac{\widehat{\varphi}_r((n-1)T_s)}{M((n-2)T_s)} \right] \quad (70)$$

Where (T_s) is the setting time, (T_r) is the rotor time constant, $\widehat{\varphi}_r(iT_s)$ and $M(iT_s)$ are respectively the estimated r rotor flux and the mutual inductance at sampling (iT_s) .

The ORFOC drive of the DSIM, given in figure (4), is initialized through applying a motor speed reference. An optimal rotor flux current by means of the deadbeat controller is delivered to the remaining part of RFOC drive. On the other hand, a transient torque current reference will be delivered to the rest of the RFOC drive.

The simulations results are carried out on a three-phase DSIM, 380V, 20KW, 50Hz and 4 poles, squirrel cage induction motor.

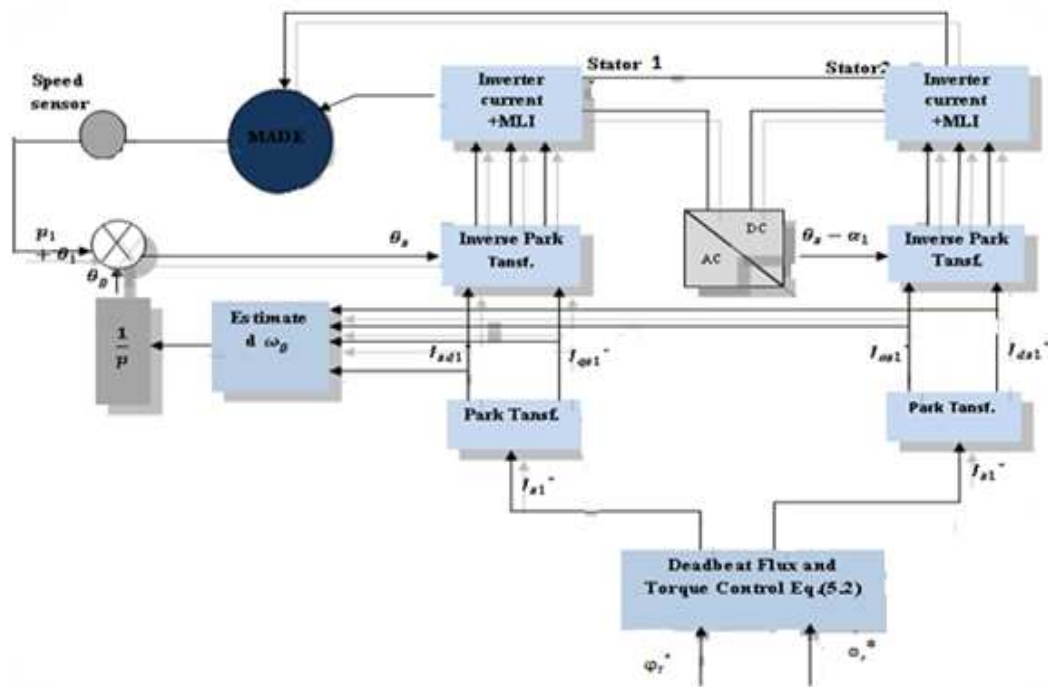


figure.4: the ORFOC drive of th DSM

9. Simulation results

By applying a rotor speed reference as shown in figure (5), a rotor flux reference is deduced from equation (68). Both the rotor flux and the torque controllers deliver to the rest of the RFOC respectively the reference of the optimal direct stator current and the reference of the desired indirect stator current reference.

It is obvious to remark from figure (6) and figure (7) that the flux current given from the conventional ORFOC remains constant during the rotor speed increasing accompanied by an increasing torque current. We show that the flux current delivered by the ORFOC registers a significant decrease compared to the one delivered by the conventional RFOC. This means that the presented method saves energy. The figure (9) shows a variation of the magnetic energy. We can see very well the decreasing of energy by adopting the ORFOC

strategy. Also, we note that this decrease is more considerable for small loads. This result given by the proposed ORFOC compared with those delivered from the conventional RFOC prove that the minimization of the cost function performed by the proposed method causes a stored magnetic energy saving and consequently a power factor maximization.

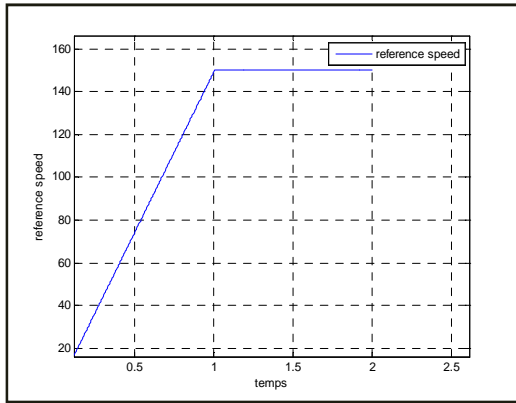


Figure 5: Reference rotor speed.

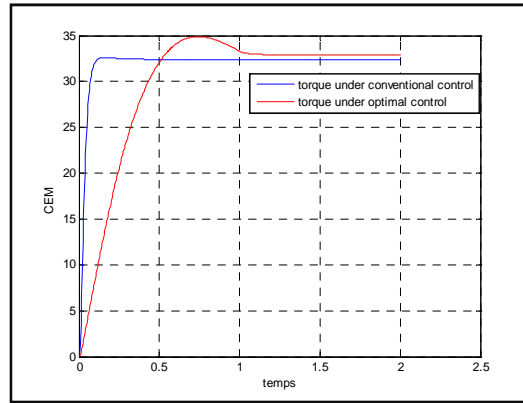


Figure 8: electromagnetic torque under conventional and optimal RFOC

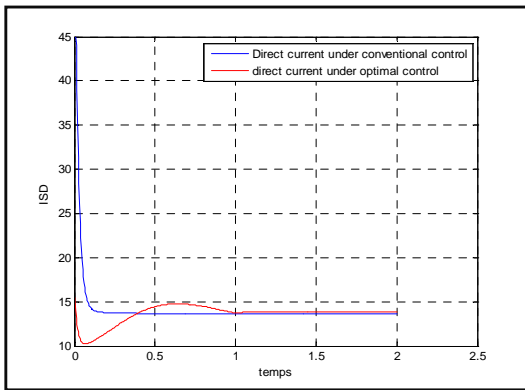


Figure 6: direct current under conventional and optimal RFOC

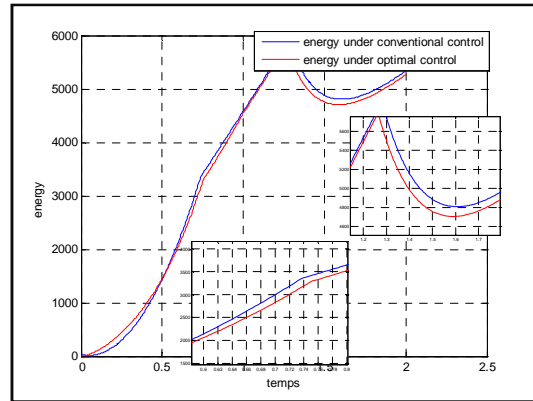


Figure 9: energy under optimal and conventional RFOC

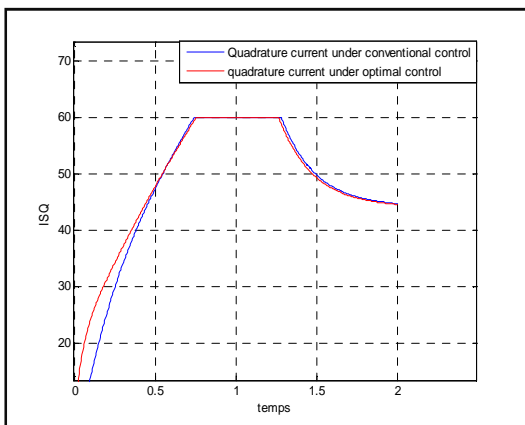


Figure 7: quadrature current under conventional and optimal RFOC

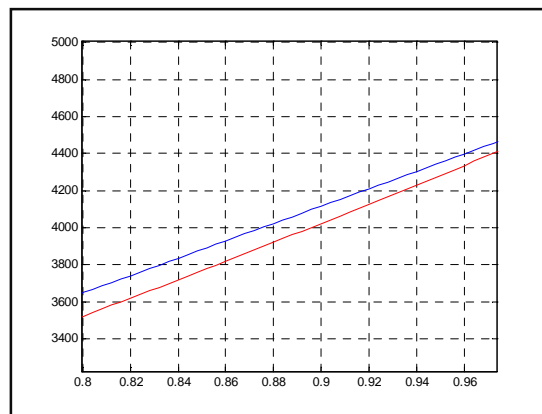


Figure 7: Zoomed Energy consumption curve.

10. Appendix

We recall the theorem of integration by parts:

$$\int_0^t \dot{U}V dt = UV - \int_0^t UV \dot{t} dt$$

$$\int_0^T (\dot{\varphi}_r \varphi_r^3 - a_2 \varphi_r^4) dt$$

$$= \int_0^T (-\gamma_1 t^2 + \gamma_2 t + \gamma_3) dt$$

This implies:

$$\dot{\varphi}(t) \varphi_r^3(t) - \varphi_r^3(t) - \frac{a_2}{5} \varphi_r^5(t) - \dot{\varphi}(0) \varphi_r^3(0)$$

$$+ \varphi_r^3(0) + \frac{a_2}{5} \varphi_r^5(0)$$

$$= - \left(\frac{\gamma_1 t^3}{3} + \frac{\gamma_2 t^2}{2} + \gamma_3 t \right)$$

This equation will be integrated again,

$$\int_0^T \left(\dot{\varphi}(t) \varphi_r^3(t) - \varphi_r^3(t) - \frac{a_2}{5} \varphi_r^5(t) \right.$$

$$\left. - \dot{\varphi}(0) \varphi_r^3(0) + \varphi_r^3(0) \right.$$

$$\left. + \frac{a_2}{5} \varphi_r^5(0) \right) dt$$

$$= \int_0^T \left(- \left(\frac{\gamma_1 t^3}{3} + \frac{\gamma_2 t^2}{2} \right. \right.$$

$$\left. \left. + \gamma_3 t \right) \right) dt$$

We obtain then:

$$-\frac{a_2 \varphi_r^6(t)}{30} - \frac{a_2 \varphi_r^6(0)}{30} - \dot{\varphi}_r(0) \varphi_r^3(0) + \varphi_r^3(0)t$$

$$+ \frac{1}{5} \varphi_r^5(0)$$

$$= - \left(\frac{\gamma_1 t^4}{12} + \frac{\gamma_2 t^3}{6} + \frac{\gamma_3 t^2}{2} \right) + kt$$

Where k is an appropriate positive constant.

This yield:

$$\varphi_r^6(t) = \frac{30}{a_2} \left(\frac{a_2 \varphi_r^6(0)}{30} - \dot{\varphi}_r(0) \varphi_r^3(0) + \varphi_r^3(0)t \right.$$

$$\left. + \frac{1}{5} \varphi_r^5(0) \right.$$

$$\left. + \left(\frac{\gamma_1 t^4}{12} + \frac{\gamma_2 t^3}{6} + \frac{\gamma_3 t^2}{2} \right) - kt \right)$$

Then:

$$\varphi_r(t) = \sqrt[6]{\frac{30}{a_2} \left(\frac{a_2 \varphi_r^6(0)}{30} - \dot{\varphi}_r(0) \varphi_r^3(0) + \varphi_r^3(0)t \right.$$

$$\left. + \frac{1}{5} \varphi_r^5(0) + \left(\frac{\gamma_1 t^4}{12} + \frac{\gamma_2 t^3}{6} + \frac{\gamma_3 t^2}{2} \right) - kt \right)}$$

11. Conclusion

In this work, a minimum-energy consumption approach is developed with a DSIM under RFOC. Based on the optimal control theory, this approach provides a cost function given as a weighted sum of the DSIM energy-power model. In order to obtain a minimum energy rotor flux trajectory, the presented task is based on minimizing this cost function when it is constrained to the dynamic equations of the rotor flux and the motor speed. By applying the Euler-Lagrange resolution, analytic solution is given at transient regime and especially for an accelerated mode of the DSIM. A minimum-energy rotor flux trajectory is then developed and implemented in the RFOC strategy.

A comparative study with the conventional RFOC is given and proves the validity of the proposed minimum-energy consumption approach.

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