

MATHEMATICAL MODELING OF POWER TRANSFORMERS

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Abstract

Single-phase and three-phase power transformer mathematical models are presented. The models allow the simulation of disturbances which cause undesirable relay operation. Simulation results are obtained to present different operating condition for the power transformer including "inrush magnetizing current, winding fault". The transient waveforms are discussed. The simulation is used to illustrate how the proposed transformer modeling benefits the power transformer protection performance evaluation. The differential equations are solved and simulated using MatLab/Simulink tool.

Key words- Power Transformer, mathematical Modeling, Magnetization inrush, winding fault.

1. Introduction

Computer simulation of power transformers and protective relays eases the burden of relay testing and relay performance evaluations. This new technology draws a lot of attention from industry, and is now becoming widely adopted [1]. The main directions in the computer modeling for the study of the power transformer electromagnetic transients are summarized in [2].

Degeneff et al. used lumped R-L-C circuit to represent transformer-winding [3]. This method requires knowledge on the details of the transformer construction to get these parameters, and these parameters are very difficult to estimate from an external testing. In [4], a method to establish a multi-section network model for study of high frequency transient behavior of the transformer and the machine winding is presented with equally divided sections. That will make the number of sections large if this method is used for simulating the small winding fault of the transformer.

Some transformer parameters are nonlinear and/or frequency-dependent due to three major effects: saturation, hysteresis and eddy currents. Saturation and hysteresis introduce distortion in waveforms; hysteresis and eddy currents originate losses. Saturation is the predominant effect in power transformers. A computationally simple way to represent transformer core nonlinear behavior is to use a piecewise linear representation [5] with multiple slopes for the B-H curve, and implemented using switched linear inductors in

several simulation programs. In this paper the hysteresis losses and eddy current losses are not considered in the modeling of the power transformer, since the simulation results presented in [6] show that the hysteresis loops usually have a negligible influence on the magnitude of the magnetizing current. The eddy currents provide appropriate damping effects in the simulation of the transients, but the eddy current damping is insignificant in the fast transient study [7].

This paper presents a simulation of mathematical transformer model with exception of the dynamic representation of hysteresis and eddy current losses. The main reason why this has been left out is the complexity of the hysteresis phenomenon, where the nonlinearity of saturation is coupled with the complicated dependence of the magnetic field intensity on the present and past values of the flux density. Also, the resistances representing eddy current losses are frequency dependent and cannot be represented by a constant resistance [8]. The MatLab/Simulink tool was chosen as computer tool for modeling the power system transformer. The paper presents the simulation of internal faults to the transformer, as well as other disturbances that can lead to a false operation of the system protection such as magnetization inrush current. The proposed model is exemplified for a 220/110 V - 5 kVA transformer.

2. The Model of Single Phase Transformer

Consider the magnetic coupling between the primary and secondary windings of a transformer shown in Figure 1 [9].

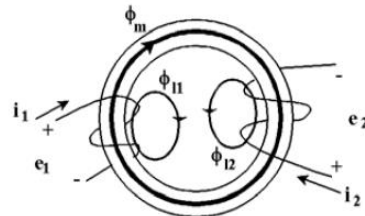


Fig. 1. Magnetic coupling of a two winding transformer.

The total flux linked by each winding may be divided into two components: a mutual component, ϕ_m , which is common to both windings and leakage flux components that link only the winding, itself. In terms of these flux

components, the total flux by each of the windings can be expressed as:

$$\varphi_1 = \varphi_{L1} + \varphi_m \quad (1)$$

$$\varphi_2 = \varphi_{L2} + \varphi_m \quad (2)$$

Where φ_{L1} and φ_{L2} are the leakage flux components of windings 1 and 2 respectively. Assuming that N_1 turns of winding 1 effectively link φ_m and φ_{L1} , the flux linkage of winding 1 is defined by:

$$\lambda_1 = N_1\varphi_1 = N_1(\varphi_{L1} + \varphi_m) \quad (3)$$

The leakage and mutual fluxes can be expressed in terms of the winding currents using the magneto-motive forces (mmfs) and permeances.

So, the flux linkage of winding 1 is:

$$\lambda_1 = N_1[N_1i_1P_{L1} + (N_1i_1 + N_2i_2) P_m] \quad (4)$$

Where $P_{L1} = \varphi_{L1}/N_1i_1$ and $P_m = \varphi_m/(N_1i_1 + N_2i_2)$.

Similarly, the flux linkage of winding 2 can be expressed as follows:

$$\lambda_2 = N_2\varphi_2 = N_2(\varphi_{L2} + \varphi_m) \quad (5)$$

And using mmfs and permeances for this winding,

$$\lambda_2 = N_2[N_2i_2P_{L2} + (N_1i_1 + N_2i_2) P_m] \quad (6)$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (7)$$

Where L_{11} and L_{22} are the self-inductances of the windings, and L_{12} and L_{21} are the mutual inductances between them. Note that the self-inductance of the primary can be divided into two components, the primary leakage inductance, L_{L1} and the primary magnetizing inductance, L_{m1} which are defined by,

$$L_{11} = L_{L1} + L_{m1} \quad (8)$$

Where $L_{L1} = N_1^2P_{L1}$ and $L_{m1} = N_1^2P_m$

Likewise, for winding 2

$$L_{22} = L_{L2} + L_{m2} \quad (9)$$

Where $L_{L2} = N_2^2P_{L2}$ and $L_{m2} = N_2^2P_m$

Finally, the mutual inductance is given by,

$$L_{12} = N_1N_2P_m \quad (10)$$

$$L_{21} = N_1N_2P_m \quad (11)$$

The voltage equations will be as following:

$$e_1 = d\lambda_1/dt = N_1d\varphi_1/dt = L_{11}di_1/dt + L_{12}di_2/dt \quad (12)$$

Replacing L_{11} by $(L_{L1} + L_{m1})$ and L_{12} by (N_2L_{m1}/N_1) the following equation can be deduced:

$$e_1 = L_{L1}di_1/dt + L_{m1}d(i_1 + (N_2/N_1)i_2)/dt \quad (13)$$

Similarly, the induced voltage of winding 2 is written by,

$$e_2 = L_{L2}di_2/dt + L_{m2}d(i_2 + (N_1/N_2)i_1)/dt \quad (14)$$

Froehlich equation describes the iron core saturation.

$$\varphi = \frac{k_1 \times i}{k_2 + i} + k_3 \times i \quad (15)$$

Finally, the terminal voltage of a winding is the sum of the induced voltage and the resistive drop in the winding; the complete equations of the two windings are,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1r_1 \\ i_2r_2 \end{bmatrix} + \begin{bmatrix} L_{L1} + L_{m1} & L_{m1}(N_2/N_1) \\ L_{m2}(N_1/N_2) & L_{L2} + L_{m2} \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \end{bmatrix} \quad (16)$$

Iron core saturation was simulated by using look up table in the Simulink tool, based on the vector of current values and the vector of corresponding magnetic flux values.

Figure 2 illustrates the Simulink model of single-phase power transformer. The sine wave represents the AC voltage source; lookup table represents the saturation of the iron core. Equations (1) – (16) are solved using the Simulink function blocks "Sum, subtract, integrator, gain, ..etc". The waveforms of " i_1 , i_2 , V_1 , V_2 , φ_m " are displayed on the scope window.

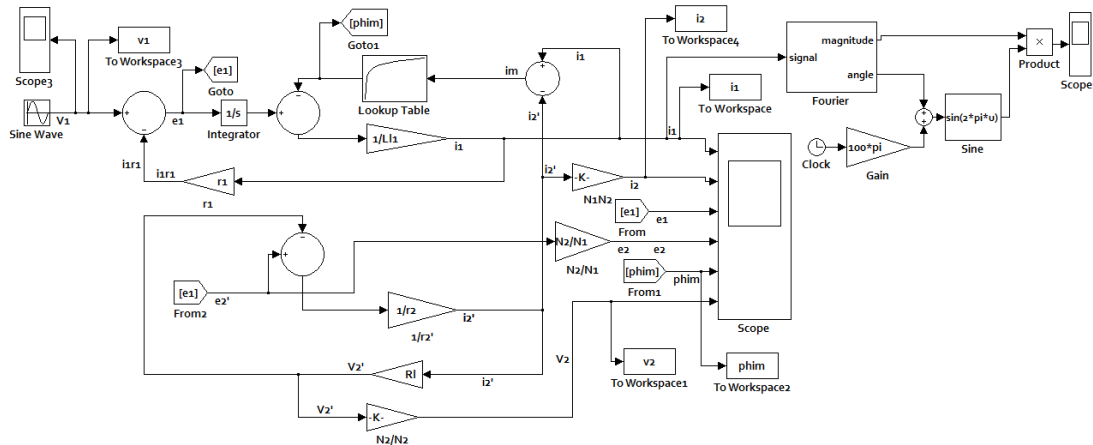


Fig. 2. Single-phase transformer Simulink model.

3. Simulation Results for Single Phase Transformer Model

3.1 Normal Operation:

In this section the figures show the instantaneous values of primary and secondary voltage and currents and the instantaneous values of effective flux in case of normal operation.

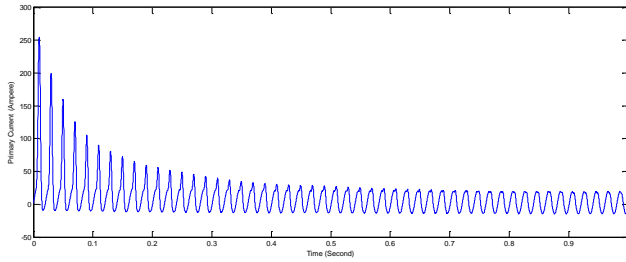


Fig. 3. The instantaneous values of primary current (i_1) during transformer switching-on.

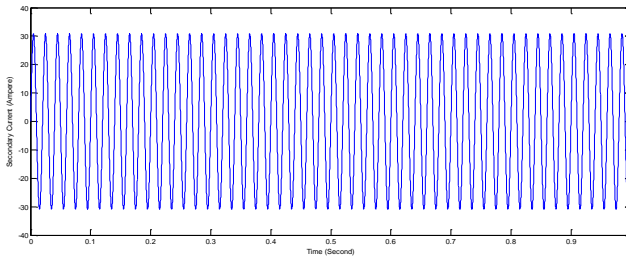


Fig. 4. The instantaneous values of secondary current (i_2) during transformer switching-on.

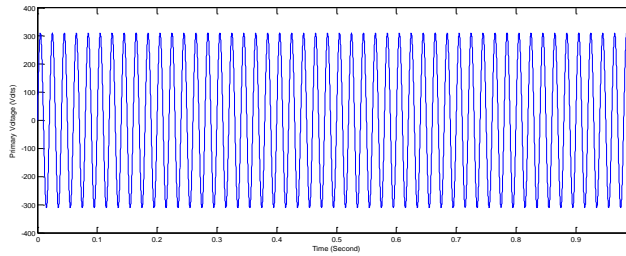


Fig. 5. The instantaneous values of primary voltage (V_1) during transformer switching-on.

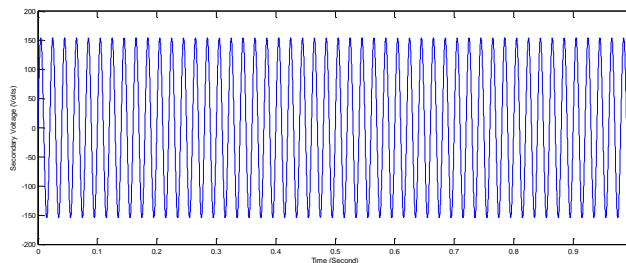


Fig. 6. The instantaneous values of secondary voltage (V_2) during transformer switching-on.

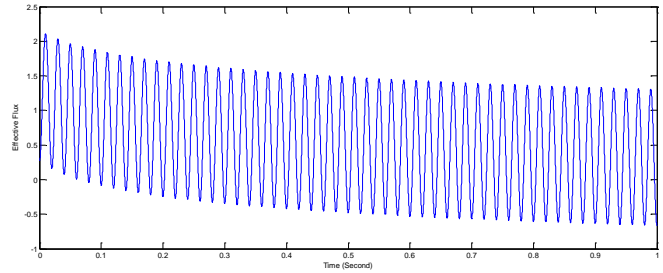


Fig. 7. The instantaneous values of effective flux (ϕ_m) during transformer switching-on.

The representation of the saturation curve is very important. It was represented in the mathematical transformer model by using look up table in the Simulink tool, based on the vector of current values and the vector of corresponding magnetic flux values.

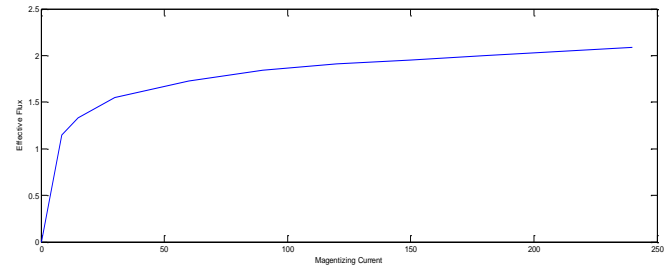
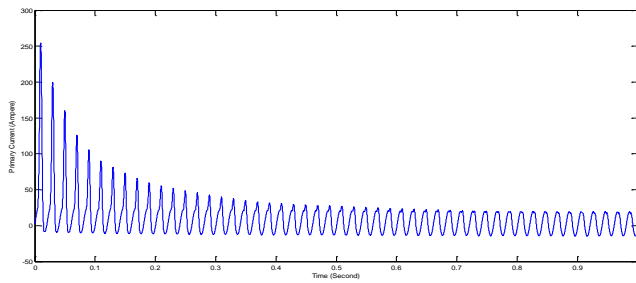


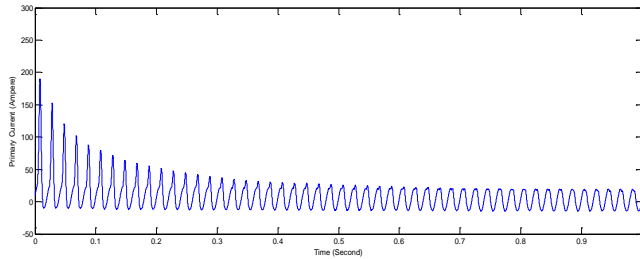
Fig. 8. Magnetization Curve.

3.2 Inrush Current case studies

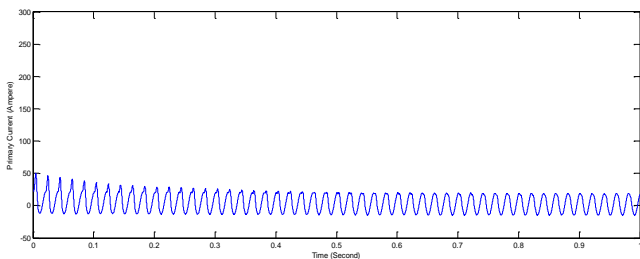
When the transformer is switched into the supply at the rated voltage with unloaded secondary, inrush currents may be observed. It is sometimes much greater than the full load current of the transformer. That phenomenon is due to the nonlinearity of magnetic circuit of the transformer. For loaded secondary transformers the inrush currents become harmful to the connected system. Although the inrush currents may reach very high values (10 times or more those of full load) [10], it is not a fault condition. Therefore the protection scheme of the transformer must remain stable during inrush transient. This requirement is a major factor in the design of the protection system. The inrush currents decay rapidly for the first few cycles and very slowly afterwards. Sometimes they take four to six seconds to subside. Because of the fast decaying of the inrush current, it is not considered a fault condition. The inrush current has two main parameters that have the most effect on its value. These parameters are the point of the voltage waveform at which switching on is occurred " θ " and the magnitude and the polarity of the residual flux " ϕ_r ". The following figures show different case studies of inrush current:



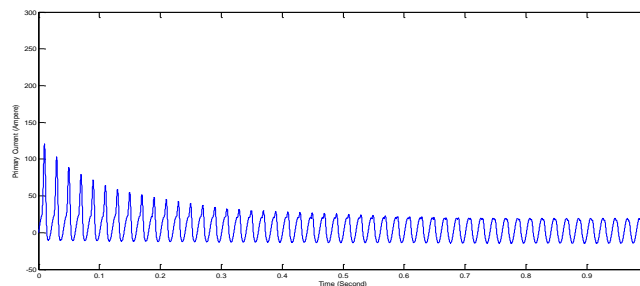
a- $\theta=0^{\circ}$, $\varphi_r=0.27$ [$i_{inrush} = 15 i_{rated}$].



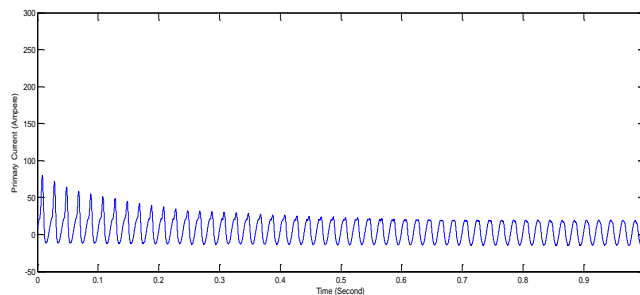
b- $\theta=30^{\circ}$, $\varphi_r=0.27$ [$i_{inrush} = 11 i_{rated}$].



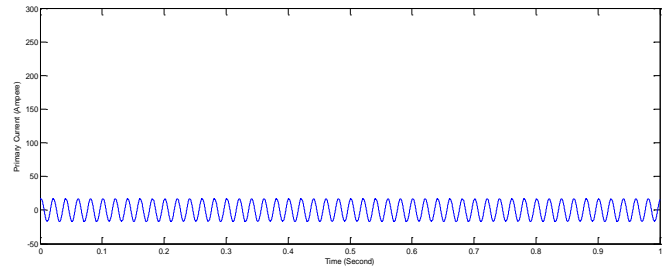
c- $\theta=90^{\circ}$, $\varphi_r=0.7$ [$i_{inrush} = 3 i_{rated}$].



d- $\theta=0^{\circ}$, $\varphi_r=0$ [$i_{inrush} = 7 i_{rated}$].



e- $\theta=30^{\circ}$, $\varphi_r=0$ [$i_{inrush} = 4.5 i_{rated}$].



f- $\theta=90^{\circ}$, $\varphi_r=0$ [$i_{inrush} = i_{rated}$].

Fig. 9. The primary current in case of switching on a transformer.

3.2.1 Fourier analysis:

It is well known that the magnetizing inrush current generally contains a large 2nd order harmonic component and large DC component [11]. The Inrush current was analyzed using Fast Fourier Transform (FFT) as shown in figures 10, 11 and 12.

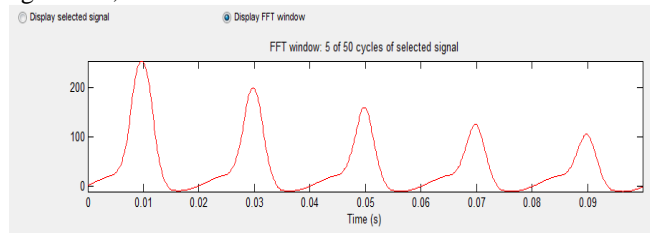


Fig. 10. Fast Fourier transform window for 5 cycles.

Total Harmonic Distortion (THD) = 65.21%

Maximum harmonic frequency used for THD calculation = 4950.00 Hz (99th harmonic)

0 Hz (DC) :	59.99%	90.0°
10 Hz	23.79%	16.6°
20 Hz	14.84%	6.3°
30 Hz	14.23%	0.0°
40 Hz	21.17%	0.0°
50 Hz (Fnd) :	100.00%	0.0°
60 Hz	17.85%	200.8°
70 Hz	10.41%	191.3°
80 Hz	9.51%	183.8°
90 Hz	13.98%	174.6°
100 Hz (h2) :	57.62%	94.8°
110 Hz	10.47%	22.7°
120 Hz	5.54%	16.5°
130 Hz	4.69%	10.7°
140 Hz	6.89%	5.9°
150 Hz (h3) :	28.76%	0.0°

Fig. 11. Fast Fourier analysis for the inrush current.

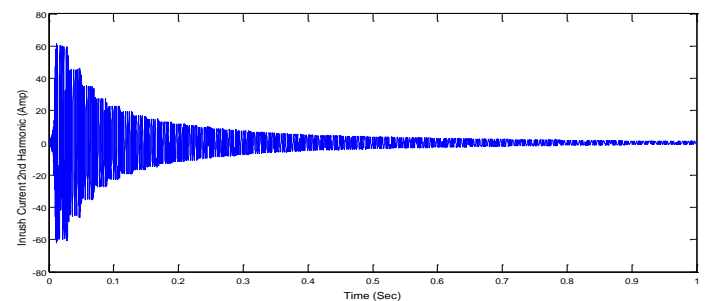


Fig. 12. Second order harmonics in Inrush current.

3.3 Winding fault:

In this section a 5 % winding fault in the secondary winding will be simulated, the fault occurred at 0.65 second (32.5 Cycle and $\theta=0^0$), the secondary winding is divided into two sections. The first is 95 % of its original winding and the second is 5 % representing the internal fault winding. For simplicity it is assumed that the reluctance is constant.

The transformer suffering from winding fault can be represented by a three winding transformer, the equations will be as following:

$$e_1 = d\lambda_1/dt = N_1 d\phi_1/dt = N_1 d(\phi_{L1} + \phi_m)/dt \quad (17)$$

$$e_2 = d\lambda_2/dt = N_2 d\phi_2/dt = N_2 d(\phi_{L2} + \phi_m)/dt \quad (18)$$

$$e_3 = d\lambda_3/dt = N_3 d\phi_3/dt = N_3 d(\phi_{L3} + \phi_m)/dt \quad (19)$$

$$\phi_{L1} = N_1 i_1 / R_{L1} \quad (20)$$

$$\phi_{L2} = N_2 i_2 / R_{L2} \quad (21)$$

$$\phi_{L3} = N_3 i_3 / R_{L3} \quad (22)$$

Where R_L is the reluctance.

$$R_{L1} = N_1^2 / L_{L1} \quad (23)$$

$$R_{L2} = N_2^2 / L_{L2} \quad (24)$$

$$R_{L3} = N_3^2 / L_{L3} \quad (25)$$

And the voltage equations will be presented as following:

$$v_1 = i_1 r_1 + N_1 d\phi_1/dt \quad (26)$$

$$v_2 = i_2 r_2 + N_2 d\phi_2/dt \quad (27)$$

$$v_3 = i_3 r_3 + N_3 d\phi_3/dt \quad (28)$$

The primary winding is connected to the same voltage source given in section (a) and the healthy part of secondary winding (95% of original secondary winding) and the

faulted winding are referred to the primary winding.

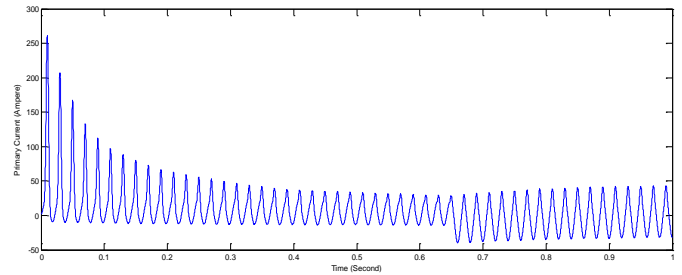


Fig. 13. The instantaneous values of primary current in case of winding fault.

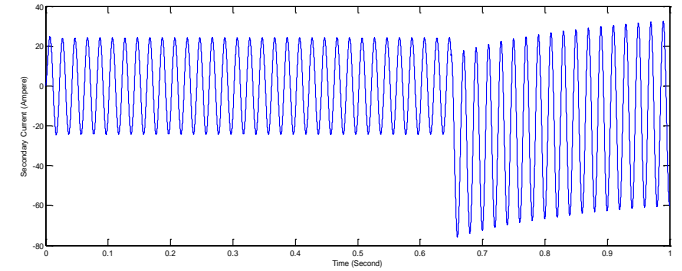


Fig. 14. The instantaneous values of secondary current in case of winding fault.

Figure 15 illustrates the Simulink model of single-phase power transformer suffering from winding fault. Equations (17) – (28) are solved using the Simulink function blocks "Sum, subtract, integrator, gain, ..etc". The waveforms of "i₁, i₂, V₁, V₂, ϕ_m " are displayed on the scope window.

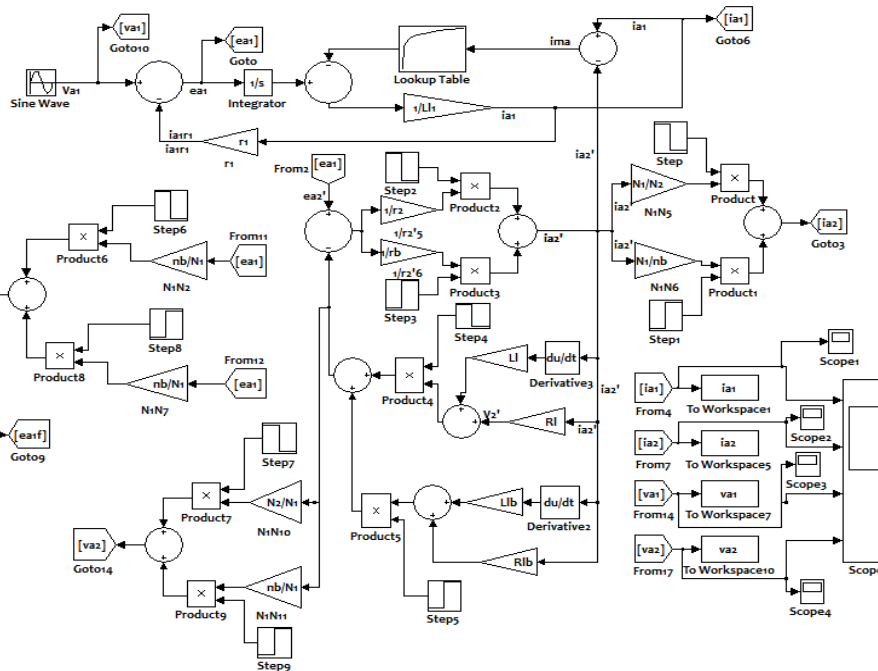


Fig. 15. The Simulink model of single-phase transformer suffering from winding fault.

3.3.1 Fourier analysis:

It is well known that the faulted current is sinusoidal and contains lower 2nd harmonics relatively to the magnetizing current [11]. The Inrush current was analyzed using Fast Fourier Transform (FFT) as shown in figure 16.

50 Hz	(Fnd) :	100.00%	0.0°
52.5 Hz		18.56%	231.3°
55 Hz		12.16%	219.3°
57.5 Hz		9.18%	211.1°
60 Hz		7.32%	205.6°
62.5 Hz		6.10%	200.9°
65 Hz		5.24%	196.9°
67.5 Hz		4.61%	194.1°
70 Hz		4.20%	192.1°
72.5 Hz		3.95%	190.2°
75 Hz		3.81%	188.6°
77.5 Hz		3.80%	187.0°
80 Hz		3.90%	185.1°
82.5 Hz		4.13%	183.1°
85 Hz		4.54%	180.3°
87.5 Hz		5.10%	175.8°
90 Hz		5.91%	170.7°
92.5 Hz		7.17%	164.6°
95 Hz		9.12%	155.3°
97.5 Hz		13.47%	141.9°
100 Hz	(h2) :	27.66%	94.0°

Fig. 16. Fast Fourier analysis for faulted current.

Since a magnetizing inrush current generally contains a large second harmonic component in comparison to an internal fault, conventional transformer protection systems are designed to restrain during inrush transient phenomenon by sensing this large second harmonic. However, the second harmonic components in the magnetizing inrush currents tend to be relatively small in modern large power transformers because of improvements in the power transformer core material.

4. Three Phase Transformer Mathematical Model

The three-phase transformer presentation is more complex than the single-phase one, by the fact that there may be an inrush in more than one-phase, these inrush currents may be affected by the electric connection and/or magnetic coupling between phases. The variant connections can be divided into three categories [12]:

- Completed separate phases
In this group the primary is connected either in Delta or in Star with earthed neutral. So, each phase behaves separately and has no effect on the other two phases.
- Star / Star connection
The primary is Star connected with isolated neutral. So, the current in each primary phase depends upon the other two phases to satisfy the relation: $i_1+i_2+i_3=0$
- Delta connected secondary
When a transformer with Delta connected secondary is switched on, in addition to transient current that flow in the primary, another transient current will circulate in the secondary winding. However, this circulating current has a helping effect to reduce the inrush current.

A three phases Star earthed – Star connection transformer bank not three phase core transformer is presented in figure 17. It should be pointed out that the latter case will be more complicated due to magnetic dissymmetry and magnetic coupling between phases. Equations (29) – (41) are solved using the Simulink function blocks, primary and secondary voltages and currents waveforms for each phase are displayed on the scope window.

The equations which represent the model are as following:

$$v_{a1} = i_{a1} r_{a1} + N_{a1} d \phi_{a1} / dt \quad (29)$$

$$v_{b1} = i_{b1} r_{b1} + N_{b1} d \phi_{b1} / dt \quad (30)$$

$$v_{c1} = i_{c1} r_{c1} + N_{c1} d \phi_{c1} / dt \quad (31)$$

$$\text{Where, } v_{a1} = V_{max} \sin (wt + \theta) \quad (32)$$

$$v_{b1} = V_{max} \sin (wt + \theta - 2\pi/3) \quad (33)$$

$$v_{c1} = V_{max} \sin (wt + \theta - 4\pi/3) \quad (34)$$

The induced voltage equations are:

$$e_{a1} = N_{a1} d \phi_{a1} / dt \quad (35)$$

$$e_{b1} = N_{b1} d \phi_{b1} / dt \quad (36)$$

$$e_{c1} = N_{c1} d \phi_{c1} / dt \quad (37)$$

The total flux for each phase can be expressed by the following:

$$\phi_{a1} = \phi_{La1} + \phi_m \quad (39)$$

$$\phi_{b1} = \phi_{Lb1} + \phi_m \quad (40)$$

$$\phi_{c1} = \phi_{Lc1} + \phi_m \quad (41)$$

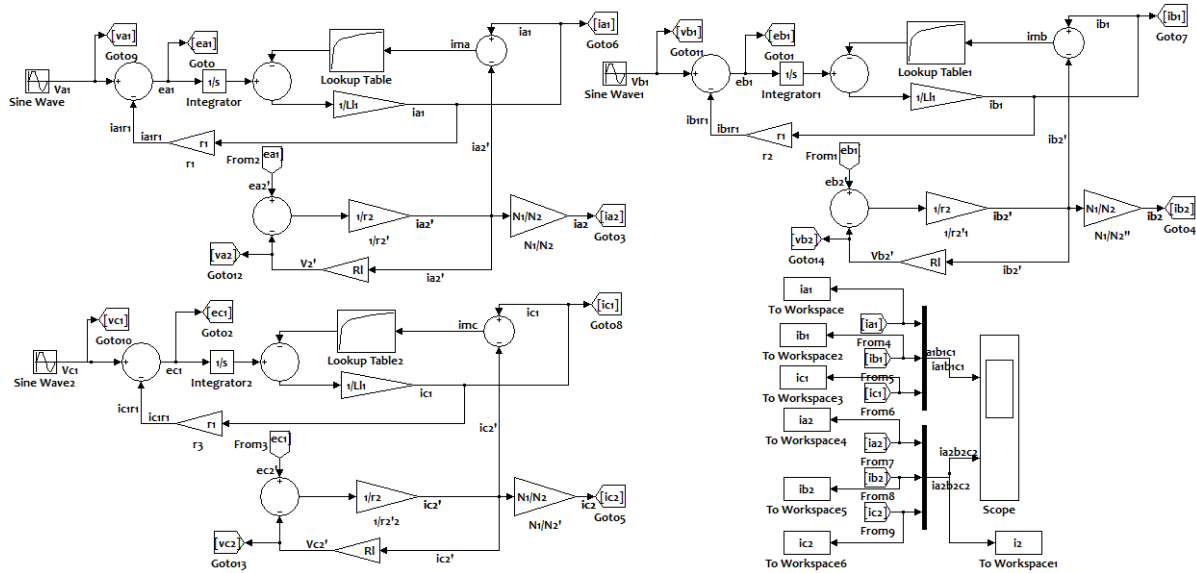


Fig.17. Three-phase transformer Simulink model.

5. Simulation Results for Single Phase Transformer Model

5.1 Normal Operation:

In this section figures 18,19,20 and 21 show the instantaneous values of primary and secondary voltage and currents of a star earthed –star three phase transformer in case of normal operation.

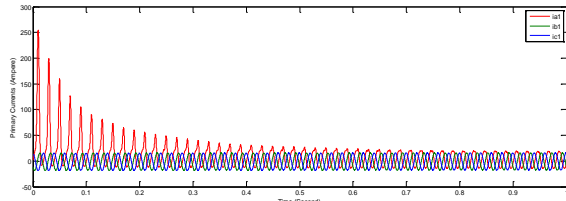


Fig. 18. The instantaneous values of primary currents (i_{a1} , i_{b1} , i_{c1}).

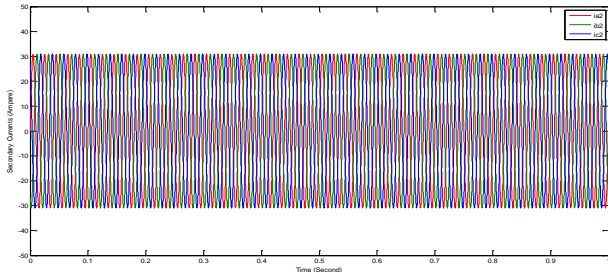


Fig. 19. The instantaneous values of secondary currents (i_{a2} , i_{b2} , i_{c2}).

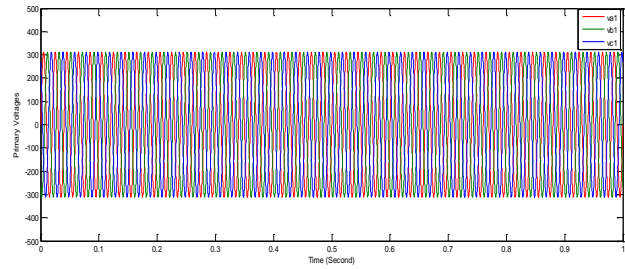


Fig. 20. The instantaneous values of primary voltages (V_{a1} , V_{b1} , V_{c1}).

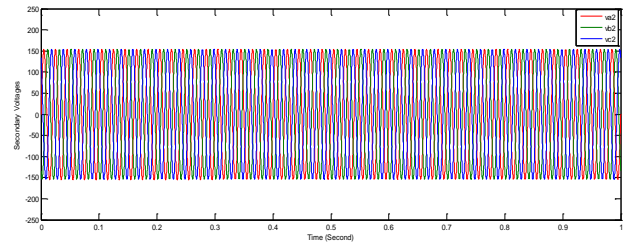


Fig. 21. The instantaneous values of secondary voltages (V_{a2} , V_{b2} , V_{c2}).

5.2 Winding fault:

In this section a 10% turn to ground fault in the secondary winding of phase (a) will be simulated, the fault occurred at 0.65 second, the secondary winding of phase (a) is divided into two sections. The first is 90% of its original winding and the second is 10% representing the internal fault winding. The equations illustrate the faulted phase is the same as mentioned in (Section 3.3). Figures 22 and 23 show the instantaneous primary and secondary currents in case of winding fault.

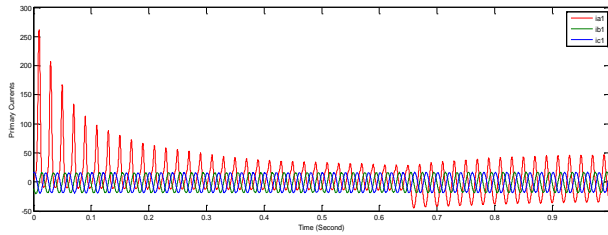


Fig. 22. The instantaneous values of primary currents (i_{a1} , i_{b1} , i_{c1}) in case winding fault.

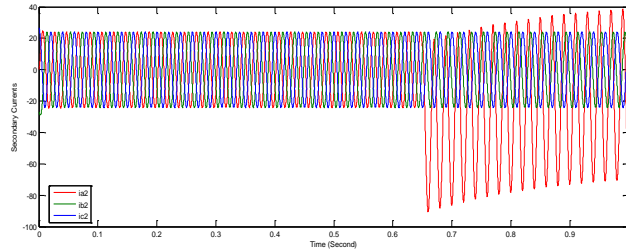


Fig. 23. The instantaneous values of secondary currents (i_{a2} , i_{b2} , i_{c2}) in case of winding fault.

6. Conclusions:

Single-phase and three-phase power transformer mathematical models were presented including the simulation of winding fault. As well as the simulation of other disturbances which may cause undesirable protection system operation.

The flux linkage and voltage equations are used to describe the transformer model. The Matlab/Simulink software package is used to simulate the developed model.

The results show the following:

The severest inrush current is obtained when the transformer is switched on at the instant of zero voltage, while the polarity of the residual flux is to assist the building up of the flux and the most favorable conditions for switching-in the transformer corresponds to switching-in at the instant the voltage is maximum with zero residual flux.

The inrush current contains decaying DC and dominant second harmonic components which may cause undesirable effects like poor power quality and reduced mean lifetime of transformer while the faulted current is sinusoidal and contains lower 2nd harmonics relatively to the magnetizing current.

The three phase transformer connection was Star Earthed/Star, so each phase behaved separately and had no effect on the other phase and the secondary had no effect.

7. Appendix

7.1 Power transformer parameters

Nominal Power = 5 kVA, frequency=50 Hz

$V_{1\text{phase rms}} = 220 \text{ V}$, $r_1 = 0.09 \ \Omega$, $L_1 = 0.3 \text{ mH}$

$V_{2\text{phase rms}} = 110 \text{ V}$, $r_2 = 0.02 \ \Omega$, $L_2 = 0.01 \text{ mH}$, $R_L = 5 \ \Omega$

Saturation Characteristic for saturable transformer = [(0,0),(8.3,1.15),(15,1.33),(30,1.55),(60,1.73),(90,1.84),(120,1.91),(150,1.95),(180,2)]

7.2 Faulted power transformer parameters

Winding one parameters: $r_1 = 0.09 \ \Omega$, $L_1 = 0.3 \text{ mH}$

Winding two parameters: $r_2 = 0.019 \ \Omega$, $L_2 = 0.0095 \text{ mH}$

Winding three parameters: $r_3 = 0.001 \ \Omega$, $L_3 = 0.0005 \text{ mH}$

$R_L = 5 \ \Omega$, $L_L = 0.125 \text{ mH}$.

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